BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

B.E. (Mechanical Engg.) Part IV 7th Semester Final Examination, 2011

Subject: Introduction to the Finite Element Method (ME-705/11)

Full Marks: 70 | Time: 3 Hours

Answer FIVE questions taking at least TWO from each group in a SINGLE answer book
Solving simultaneous equations directly using a CALCULATOR is allowed

Group - A

1a) Starting from the expression of the potential energy, derive the equation of a 2-node bar element that have concentrated forces at its ends and is subjected to a temperature change of ΔT. Other material properties of the element are modulus of elasticity E and coefficient of linear expansion α. Length of the element is L and cross-sectional area is A.

b) Consider the bar shown in Fig. 1, subjected to a force of 300 kN and a temperature rise of 60°C. Determine the displacement at node 2. Properties are as follows:

Material	E (GPa)	A (mm^2)	α (/°C)
Aluminum	70	900	23×10^{-6}
Steel	200	1200	11.7×10^{-6}

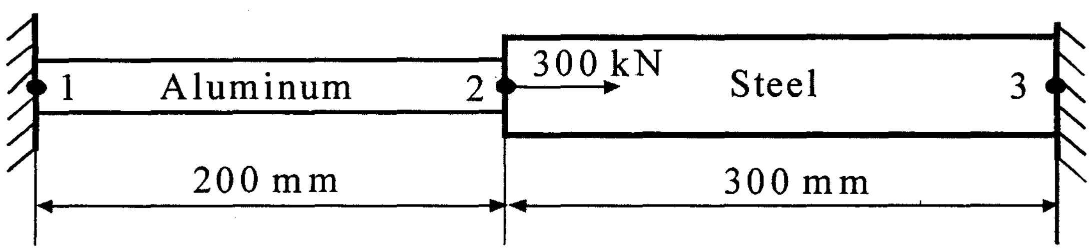

Fig. 1

[6+8 = 14]

2) The following boundary value problem is to be solved by the finite element method:

$$-\frac{d^2u}{dx^2}+x^2=0 \text{ in } 0<x<1, \text{ subject to } u(0)=0 \text{ and } \left(\frac{du}{dx}\right)\bigg|_{x=1}=1$$

Obtain the weak form of the weighted integral statement of the given equation. Now from the weak form, derive the equation for a 2-node finite element either by the Galerkin procedure or by constructing the functional for the problem. Discretize the domain by two identical linear elements and obtain values of u at x=0.5 and at x=1.0. [14]

3a) How the axial load and deformation modeling aspects are incorporated in a conventional beam element? State briefly the changes that take place in the degrees of freedom and the associated stiffness matrix.

b) The bar element 2 is used to stiffen the cantilever beam element 1, as shown in Fig. 2. Calculate all the displacement components of node 1. For the bar, A = 1.0e-03 m^2. For the beam, A = 2.0e-03 m^2, I = 5.0e-05 m^4, and L = 3.0 m as shown. For both the bar and the beam elements, E = 210 GPa. Symbols have their usual meaning. [4+10 = 14]

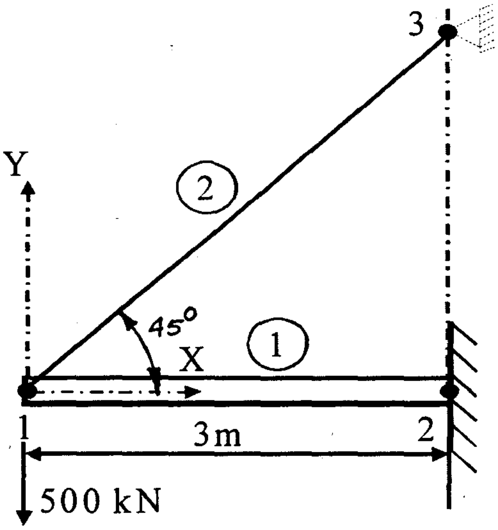

Fig. 2

The stiffness matrix of a typical beam element of length L and flexural rigidity EI may be taken as

$$[k_e]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

4a) The coordinates of the vertices of a three node triangular element according to its connectivity are (1.5, 2), (7, 3.5) and (4, 7) respectively. P is a point interior to the element, with the x-coordinate of 3.85. The first area-coordinate (L_1) of point P is 0.3. Calculate the other two area coordinates of P and its y-coordinate.

b) For the element stated in part (a), derive its Jacobian considering area coordinates as the natural coordinates. Show that the determinant of the Jacobian is twice the area of the triangle. Using the elements of the Jacobian, derive the strain displacement matrix [B] for the element and show that it is constant. [4 + 10 = 14]

Group B

5a) Determine the nodal forces for the quadratic varying pressure ($p_x = P_0 \frac{y^2}{L^2}$) on the edge of the 3-node triangular element as shown in Fig. 3. Assume the element thickness to be t.

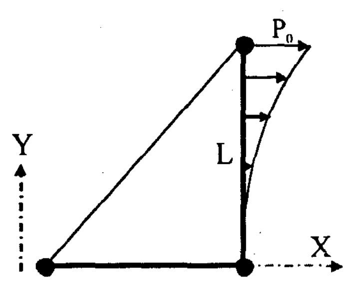

Fig. 3

b) For the plane strain elements shown in Fig. 4, the nodal displacements are given as $\{u^e\} = [0.005, 0.002, 0, 0, 0.005, 0]^T$ mm. Determine the element strain and stress components. Use E = 70 GPa, $\upsilon = 0.3$ and unit thickness. [4 + 10 = 14]

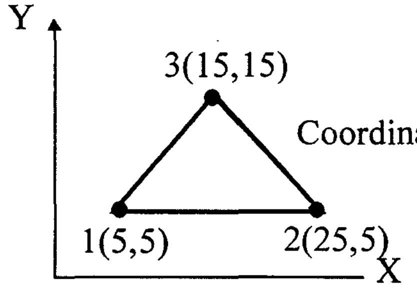

Fig. 4

6) Derive the equation of a 3-node triangular element for a 2-D heat transfer problem given by $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Q=0$. The coordinates of the nodes, in the order of connectivity, are (-2,-2), (4,0) and (0,6) in meter respectively. The edge i-k coincides with the convective boundary of the domain with a convection coefficient h = 20 W/(m^2-°C). Thermal conductivity and ambient temperature are k = 15 W/(m-°C) and T_a = 15°C respectively. The heat generation rate Q = 100 W/m^3. The other two edges of the element are internal to the domain. Thickness of the element may be taken as 1m. [14]

7a) What do you understand by a non-conforming element? Can they be used in FE formulation? If yes, how? [2+1+3 = 6]

b) A 4-node quadrilateral element has coordinates (1,1), (5,1), (6.6) and (1,4), in order of connectivity in the global domain. Using 2x2 Gauss-Quadrature formula evaluate the integral $\iint_A (x-y)dxdy$, where A denotes the area of the element. Take W(1) = W(2) = 1.0; $r_{1,2}$ = ±0.577; $s_{1,2}$ = ±0.577. Symbols have their usual meaning. [8]

8a) Write briefly the modifications required in the global stiffness matrix of a plane truss if it has a roller support inclined to the global axes.

b) For the plane truss supported by the spring at node 1, shown in Fig. 5, determine the displacement components of node 1. Take E = 210 GPa, A = 5.0e-4 m^2 for both the truss elements. The spring stiffness is 2000 kN/m. [4+10 = 14]

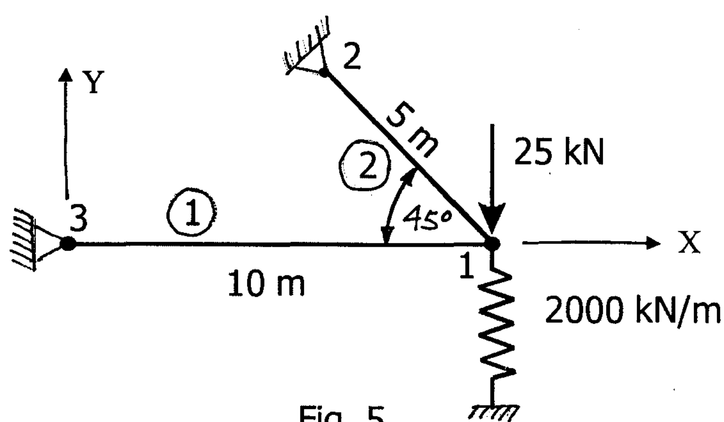

Fig. 5