B.E. (ME) Part-Ill 6th Semester Examination, 2010

Numerical Methods in Engineering
(ME-605)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.
Answer SIX questions, taking THREE from each half.
The questions are of equal value.

FIRST HALF

1. a) In which case the LU- Decomposition method has a clear advantage over other methods in solving a set of linear algebraic equations? State reasons.

   b) Solve the following equation set by the Doolittle version of the LU-Decomposition method. Employ partial pivoting wherever necessary,

   $x_1 - 2x_2 + 6x_3 = 0$

   $2x_1 + 2x_2 + 3x_3 = 3$

   $-x_1 + 3x_2 = 2$

   (No credit will be given if partial pivoting is not employed).

2. a) Following system of equations evolved to determine concentrations (c in gm/m^3) in a series of coupled reactors :

   $-5c_1 - 5c_2 + 22c_3 = 30$

   $17c_1 - 2c_2 - 3c_3 = 500$

   $-5c_1 + 21c_2 - 2c_3 = 200$

   Show two iteration steps with relaxation of the Gauss-Seidei to solve the above equation set. Take relaxation factor = 0.9. State the solution strategy and reasons for it. Initial guess may be aken as cj = C2 = c_3 = 0.0.

   b) In cubic spline interpolation, the cubic polynomial in any i^{th} interval may be expressed as,

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   where **Xj,** y; are the x, y values of any i^{th} data and h, is the length of any i^{th} interval.

y_i'' = 2^{nd} derivative of $f_i(x)$ evaluated at x_i. The unknown y" are evaluated from equation set obtained by satisfying first derivative continuity of cubic functions of adjacent intervals at the connecting point. Show that the equation set to solve for y_2'' through y_{n-1}'' for a natural cubic spline may be expressed as :

$2(h_1 + h_2)$ h_2 y_2

h_2 $2(h_2 + h_3)$ h_3 y_3 ;

h_{n-3} $2(h_{n-3} + h_{n-2})$ h_{n-2}

h_{n-2} $2(h_{n-2} + h_{n-1})$ y_{n-1}

$$= 6 \begin{bmatrix} \frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1} \\ \frac{y_4 - y_3}{h_3} - \frac{y_3 - y_2}{h_2} \\ \vdots \\ \frac{y_n - y_{n-1}}{h_{n-1}} - \frac{y_{n-1} - y_{n-2}}{h_{n-2}} \end{bmatrix}$$

where n is number of data.

3. Growth of bacteria count (y) in a culture after x hours is given in the following list

x :	0	1	2	3	4	5	6
y :	32	47	65	92	132	190	275

Obtain a fit for the above data in the least square sense using the approximating function $y = ab^x$, giving equal weights to the errors in y. If you use any formula, then prove it. Estimate y at $x = 7$.

4. a) Briefly outline the classic 4^{th} order Runge-Kutta algorithm.

b) A spherical water tank of Radius R is drained through a circular orifice of radius r at the bottom of the tank. The governing equation giving height (h) of water level at any time t is given by

dt $2hR - h^2$

$r = 40$ mm

$g = 9.8 m/s^2$

At time $t = 0$; $h_0 = 6.9$m.

Obtain h at $t = 7.5$ sec using a step size of 7.5 sec by the classic 4^{th} order Runge-Kutta method.

(ME-60S)

5. Use the non-self starting type modified Euler method to obtain y and $\frac{dy}{dx}$ at $x = 0.4$ for the initial value problem

$$\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y^2$$

subject to $y(0) = 1$ and $\frac{dy}{dx}(0) = 2$

Resolve the 2nd order equation into two 1st order simultaneous equations and solve.

Take step size $h = 0.2$ and employ single correction.

SECOND HALF

6. a) Find a real root of the equation $x - \cos x = 0$ using Newton-Raphson method, correct to three decimal places.

   b) Find a positive root of the equation $x^3 + 2x - 5 = 0$ between 1 and 3 using the method of false position correct to three decimal places.

   c) Show that the error introduced in Trapezoidal rule is proportional to the square of the length of each sub-intervals. [4+4+3'/i]

7. a) Perform three steps of Graeffe's root-squaring method on the following polynomial and find the corresponding solutions.

$$x^3 - 7x^2 + 10x - 2 = 0$$

   b) In the following table values of y for consecutive values of x are given. Find the polynomial which approximates these values using Newton's forward difference interpolation formula. Also find the value of y for x = 6.

x	3	4	5	6	7
y	13	21	31	43	57

8. a) Solve the following system of non-linear equation applying Newton-Raphson method, correct to three decimal places.

$$x^2 = 3xy - 7, \quad y = 2(x + 1)$$

   b) The population of a town in decennial census were as under. Estimate the population for the year 1995 using Lagrangian interpolation method.

Year	1961	1971	1981	1991	2001
Population (in thousands)	46	66	81	93	101

9. a) Use the table given in Q.8.(b) and find the rate of population growth for the years 1971 and 2001.

b) Solve the following boundary-value problem with $h = 0.5$ and $h = 0.25$ over the interval $[0,1]$ and comment on the effect of h on solution. [5+6½]

$$y'' - 64y + 10 = 0$$
$$y(0) = y(1) = 0$$

10. a) Compute the values of the following integral with $h = 0.5$, $h = 0.25$ and $h = 0.125$.

Now obtain a better estimate using Romberg's method. Compare all the results with the analytical solution.

b) Prove the essential criterion for convergence of the method of iteration.

[7+4½]