B. E. (Mech.) Part-II 3rd Semester Examination, 2011

Dynamics of Rigid Bodies (AM 303)

Time: 3 Hrs. Full Marks: 70

Use separate answer scripts for each half
Answer six questions taking three from each half
All questions are of equal value
Two marks are reserved for neatness in each half

FIRST HALF

1. a) The elements of a wheel-and-disk mechanical integrator are shown in the figure Fig. Q. 1(a). The integrator wheel A turns about its fixed shaft and is driven by friction from disk B with no slipping occurring tangent to its rim. The distance y is a variable and can be controlled at will. Show that the angular displacement of the integrator wheel is given by $z = (1/b)\int y dx$, where x is the angular displacement of the disk B.

   b) Using vector concept prove that during a 3D motion of a rigid body with one point fixed at any instant the body may be considered rotating about an instantaneous axis passing through the fixed point.

2. a) The wheel rolls on the circular surface without slipping. In the bottom position, it has an angular velocity ω and an angular acceleration α, both clockwise. For the position shown, obtain expressions for the acceleration of point C on the wheel in contact with the path and for the acceleration of point A. (Fig. Q. 2. (a))

   b) Figure Q. 2. (b) shows a rigid but light wire bent in the way shown. It carries three identical particles of mass '*m*' each. Each sector of the bent wire is also of equal length '*a*'. The point '*O*' is fixed and the body is rotated at a velocity $\varpi \left(= \hat{i}\ 0.707\omega + \hat{j}\ 0.707\omega\right)$. Find out the angular momentum about *O*.

3. a) Determine the angular velocity of the telescopic link AB for the position shown where the driving links have angular velocities indicated. (Fig. Q. 3. (a))

   b) Figure Q. 3. (b) shows a uniform circular disc of mass m and radius r resting on an inclined plane. The coefficient of friction between the disc and the inclined plane is μ. Find out the maximum value of θ so that the disc rolls without slipping when released.

4. a) The wheel rolls on the circular surface without slipping with angular velocity and acceleration ω and α respectively. Show that the acceleration of the instantaneous center of zero velocity is independent of α and is directed toward the center of the wheel. (Fig. Q. 4. (a))

   b) For stopping the spin of a rotating axisymmetric satellite two identical particles of mass m each are ejected from the centre by two springs of stiffness *k*, each, through the smooth passages as shown. When the particles leave the satellite the spin stops. The moment of Inertia of the satellite is *I* and the radius is *R*. Find out the compression required for each spring. The original angular speed is Ω. (Fig. Q. 4. (b))

SECOND HALF

5. a) An experimental vehicle A travels with constant speed v relative to the earth along a north-south track. Determine Coriolis acceleration a_{Cor} as a function of the latitude θ. Assume an earth fixed rotating frame Bxyz and a spherical earth. If the vehicle speed is v = 500 km/h, determine the magnitude of the Coriolis acceleration at (a) the equator and (b) the North Pole. (Fig. Q. 5. (a))

b) A wheel of external radius *R*, mass *m* and moment of inertia I is resting on a horizontal rough plane. The hub of the wheel has a radius *r* that is equal to I/mR (Fig. Q. 5. (b)). A rope is wound around the hub and the free end of the rope is pulled with a force *F* as shown, the straight portion of the rope being horizontal. Find out the acceleration of the wheel centre *O*.

6. a) Figure Q. 6 (a) shows a crushing mill schematically. The roller has mass of 1000 kg. The central vertical shaft is rotated at 600 rpm. Find out the magnitude of the crushing force.

b) A circular wheel of radius 0.1 m is hung by a rope as shown in Figure Q. 6 (b). The rope is wound around the wheel and when releases, the wheel rolls down due to gravity. If the mass of the wheel be 10 kg and moment of inertia be 0.08 kg-m^2 find out the minimum required tensile capacity of the rope so as to prevent its tearing. (Fig. Q. 6. (b))

7. a) The three principal moments of inertia of an object are I_1, I_2 and I_3. Prove that if the body is free from all external forces and moments then the body's motion is stable if it is given an initial rotation about the principal axis for which the moment of inertia is either maximum or minimum out of the three. <u>You need not derive</u> Euler's equation.

b) Using self explanatory neat sketches explain the principle of working of a gyrocompass. Use modified Euler's equation.

8. a) A space station may be considered as an axisymmetric body for which the moment of inertia about the axis of symmetry is double that about any other mutually perpendicular axis. For stabilization and for creating artificial gravity the space station is given a rotation about the symmetry axis at the rate of 3 rev/min. If it is slightly disturbed so that the angular momentum vector and the symmetry axis gets inclined at a very small angle show that the symmetry axis precesses about the angular momentum vector at a rate – 6 rpm. Derive the relation you use.

b) Describe the principle of passive directional stabilization of terrestrial satellites.

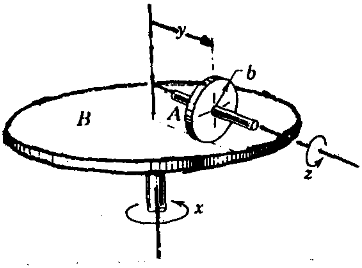

FIG Q.1.(a)

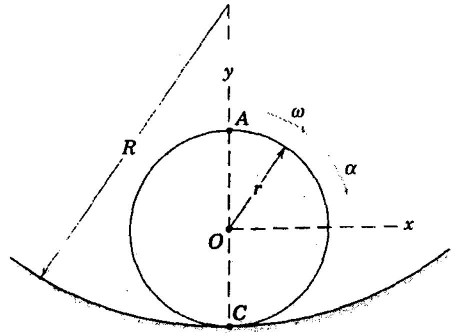

FIG. Q.2.(a)

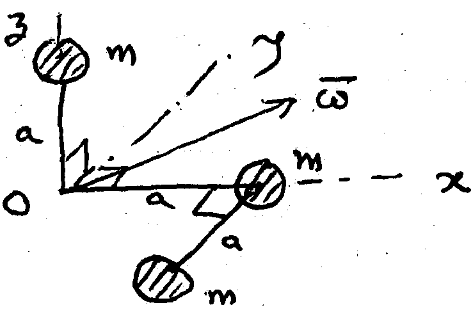

FIG. Q. 2(b)

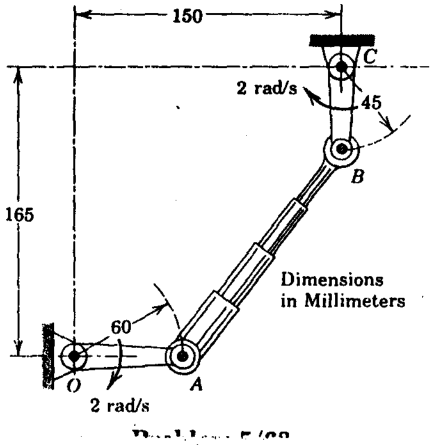

FIG Q. 3(a)

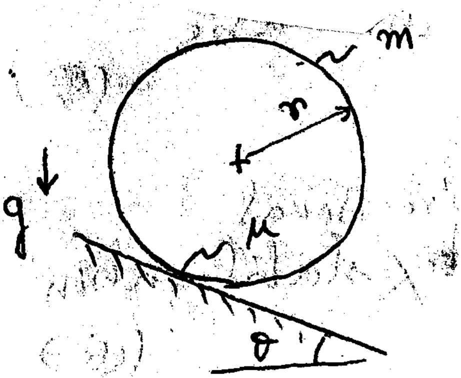

FIG Q. 3(b)

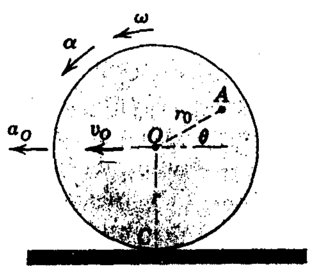

FIG Q. 4(a)

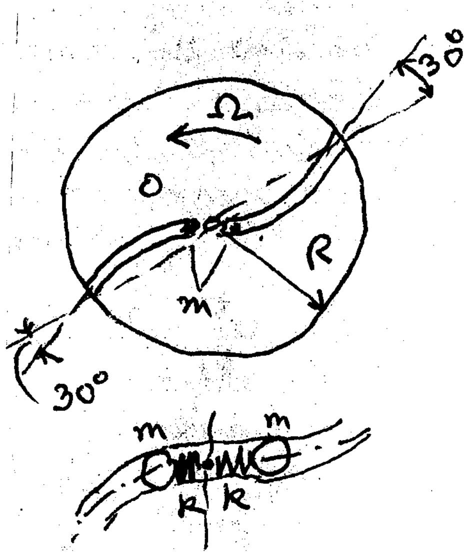

FIG Q. 4(b)

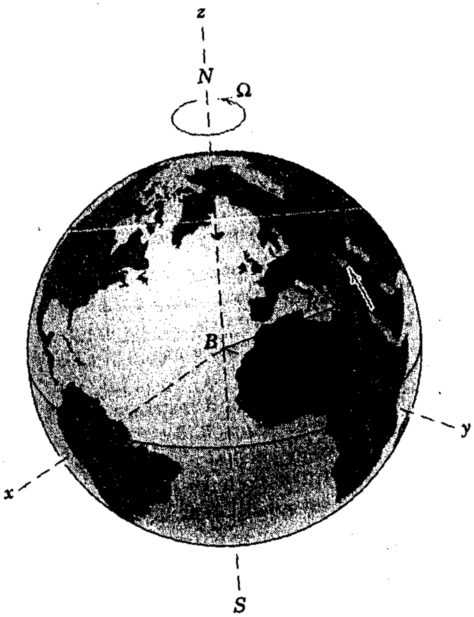

FIG Q. 5 (a)

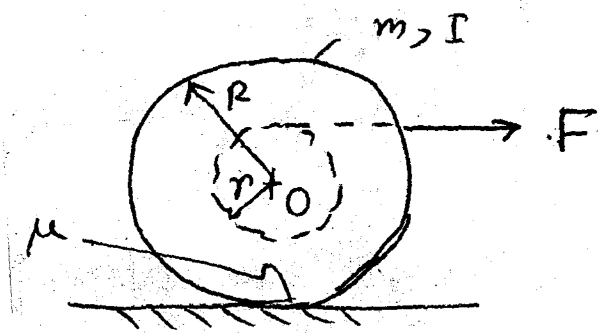

FIG Q. 5 (b)

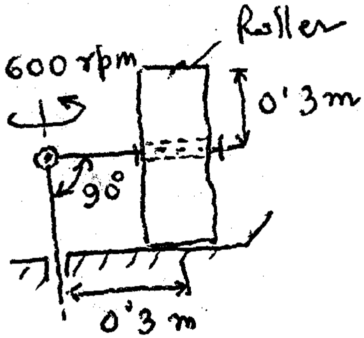

FIG Q. 6 (a)

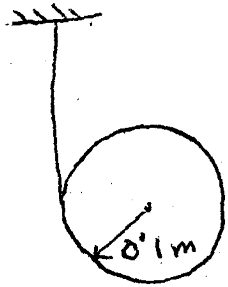

FIG Q. 6 (b)