BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

B.E. (Mechanical Engg.) Part IV 7th Semester Examination, 2013

Subject: Introduction to the Finite Element Method (ME-705/11)

Full Marks: 70 | Time: 3 Hours

Answer **1 OR 2**, **3 OR 4**, **any TWO from 5-7** and **8**

Write answers in a single answer-script

Obtaining solution of simultaneous equations directly using a CALCULATOR is allowed

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1a) Show that the stiffness matrix of a plane truss element can be expressed as

$$[k_e] = \frac{AE}{l}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix};$$

where, $c = \cos\alpha$, $s = \sin\alpha$, α being the angle of inclination of the truss member with the +x axis. Other symbols have usual meaning.

b) For the roof truss shown in Fig. 1, compute the stiffness matrix of element 3 and position the elements of this matrix in the global stiffness matrix of the entire truss. Nodal connectivity of element 3 is (4, 2). No other calculation is necessary. [7+7=14]

2a) Using the minimum potential energy approach, show that the element equation of an 1-D bar element can be expressed as $\begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$, symbols have their usual meaning.

b) For the bar assemblage of two bars and one spring shown in Fig. 2, determine the nodal displacements at nodes 2 and 3, force induced in element 2 and the support reactions. [5+9=14]

3a) State the conditions to be fulfilled in choosing trial functions while seeking approximate solutions by the weighted residual method for ODEs defining boundary value problems.

b) State how the weighting functions are chosen for the Least-square procedure and the Galerkin procedure.

c) A boundary value problem is defined as $\frac{d^2y}{dx^2} + y = 2\sin x$ in 0<x<1, subject to $y(0) = 0, y(1) = 0$. Obtain a two term approximate solution of the above using the Galerkin procedure. Choose trigonometric trial functions and use the strong form of the equation.

[3+3+8=14]

4) A circular rod has an outside diameter of 60 mm, length of 1m and is perfectly insulated on its circumference. The left half (length of 0.5m) of the rod is aluminum, for which thermal conductivity k = 200W/m-°C and the right half is copper for which k = 389 W/m-°C. The extreme right end of the rod is maintained at 80°C, while the left end is subjected to a heat input rate 4000W/m^2. Derive the governing equation of the problem at the steady state, obtain the weak form and element (linear type) equation. Using two equal linear elements, calculate the steady state temperatures at the left end and the middle point of the rod.

[14]

5) A 4-node isoparametric quadrilateral element has its coordinates as (0,0), (2,0), (1,3), (0,2) taken in the same order as its connectivity in the mesh. Is it a conforming element? Give reason in support of your answer. To evaluate a source term it is necessary to evaluate $Q\int_{\Omega^e} xy d\Omega$, where Q is a constant and Ω^e is the domain of the above quadrilateral. Transform the above integral in the parent domain and evaluate using 2x2 Gauss quadrature formula. Take $w_{1,2}$ = 1.0, $r_{1,2}$= ±0.57735 and $s_{1,2}$= ±0.57735. Symbols have usual meaning. [14]

6a) What do you understand by conformity and completeness?

b) Derive shape functions N_2 and N_4 for a six node triangular element in terms of area coordinates and then show that $\int_{A^e} N_2 N_4 dA = 0$.

c) The functional of a 1-D boundary value problem is expressed symbolically as $\pi = \int_{x_i}^{x_j} F(x,u,u_x,u_{xx})dx$, x being the independent variable and u_x and u_{xx} are the first and second derivatives of dependent variable u with respect to x respectively. Show that minimization of π (first variation = 0) will lead to the governing equation of the problem as $\frac{\partial F}{\partial u} - \frac{d}{dx}\left(\frac{\partial F}{\partial u_x}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial u_{xx}}\right) = 0$. [2+5+7=14]

7a) Plot the natures of the shape functions of a beam element with proper markings. No derivation is necessary.

b) For the beam and loading shown in Fig. 3 determine the slopes of the deflected beam at the propped ends. Use the following and symbols have usual meaning.

$$[k_e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}; \quad \{f_e\}_q = \begin{Bmatrix} qL/2 \\ qL^2/12 \\ qL/2 \\ -qL^2/12 \end{Bmatrix}$$

[4+10=14]

8a) Why is a 3-node triangular element called a CST element?

b) The Poisson equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2$ is valid in the domain with boundary conditions u=0 on Γ_1 and $\frac{\partial u}{\partial n} = 0$ on Γ_2 as shown in Fig. 4. Derive the element equation for a 3-node triangle from the weak form of the above equation using the Galerkin procedure. Now compute the stiffness matrix of element 2 of the shown mesh and position its elements in the global stiffness matrix. Connectivity may be taken as 5-4-2. [2+12=14]

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Figures of *Introduction to the Finite Element Method (ME-705/11)*

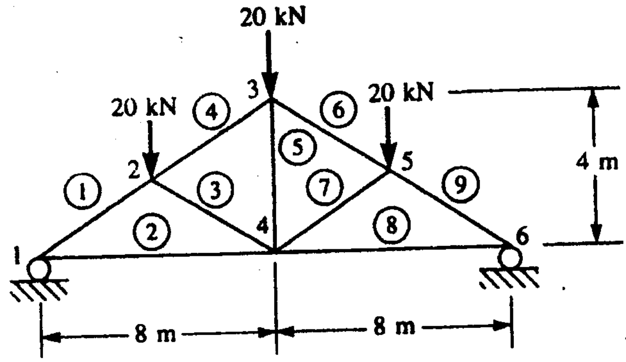

Fig. 1

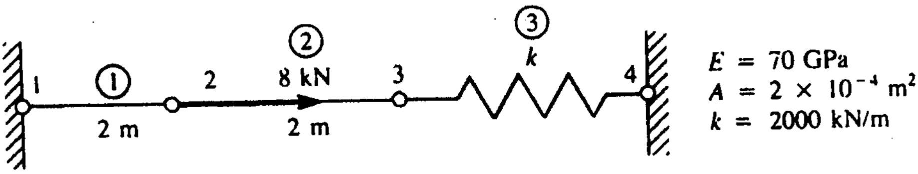

Fig. 2

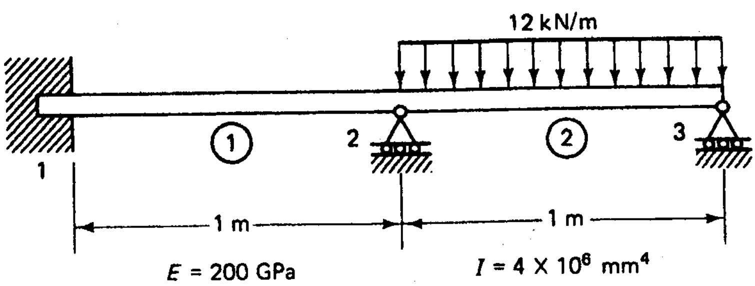

Fig. 3

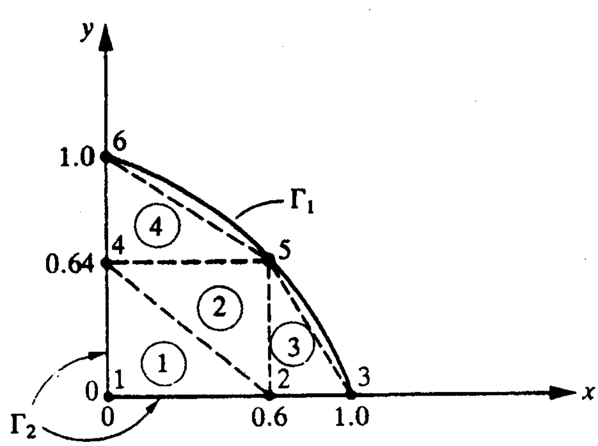

Fig. 4