BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

**B.E. (Mechanical Engg.) Part IV 7th Semester Examination, 2012**

**Subject: Introduction to the Finite Element Method (ME-705/11)**

Full Marks: 70 Time: 3 Hours

**Answer Q No. 1 and Four others taking any TWO from each Group**

Write answers in a single answer-script

Obtaining solution of simultaneous equations directly using a CALCULATOR is allowed

Assume any other data, not given but may be necessary

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1a) State two major advantages of weak form over strong form in the weighted residual method.

b) Show that the sum of shape functions of a 4-node rectangular element (parent of any quadrilateral element) in natural coordinates is 1, anywhere within the element.

c) A quadratic polynomial is given by $P(x,y) = a_0 + a_1x^2 + a_2xy + a_3y^2$. Is this function suitable for representing the field variable in a four-node rectangular element having 1 degree of freedom per node? Give reasons in support of your answer in not more than two sentences.

d) Derive the shape function $N_4$ of a 6-node triangular element in terms of area coordinates and calculate $\int_A N_4^2 dA$. Node 4 is at the mid-point of the side containing nodes 1 and 2.

(e) Does the formulation of a beam problem using a 2-node element conform to the isoparametric formulation? State your reason in not more than two sentences.

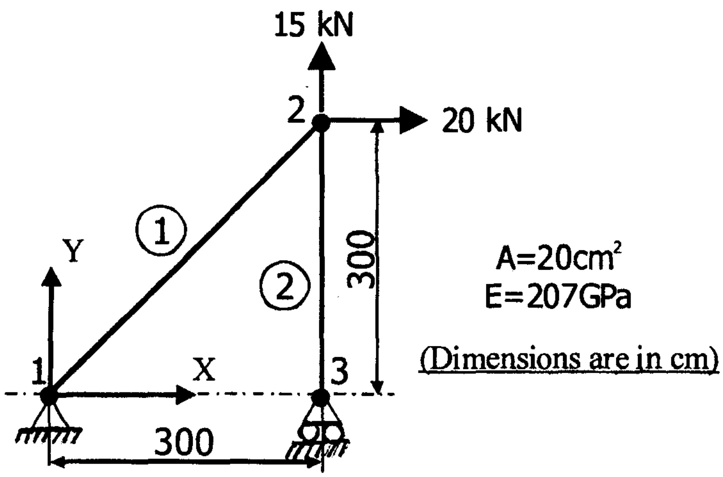
(f) Compute the integral $\int_\Omega N_1^2 d\Omega$ for a 4-node quadrilateral element. The integration has to be done using the 2 by 2 Gauss-Quadrature formula, with the Gauss points located (in the parent element) at $r_{1,2}= \pm 0.57735$ and $s_{1,2}= \pm 0.57735$ respectively. Take Jacobian [J] for the element as $\begin{bmatrix} 1.5 & 0.60566 \\ -0.5 & 1.60566 \end{bmatrix}$. [2+2+2+2+2+4=14]

## Group - A

2a) Using the principle of minimum *functional*, show that the equation of a 1-D linear spring element of stiffness k can be expressed as $\begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$, symbols have usual meaning.

b) Compute the displacement components at nodes 2 and 3 for the plane truss shown in Fig. 1 using the direct stiffness method. Obtain the global equation set and state the boundary conditions. The stiffness matrix of a plane truss element may be taken as

$$[k_e] = \frac{AE}{l} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$; symbols have usual meaning.




[3+11=14]

Fig. 1

3) For the boundary value problem given by the ODE $\frac{d}{dx}\left[(1+x)\frac{du}{dx}\right]=0$, subject to u(0) = 0 and u(1) = 1, obtain a two term solution by the weighted residual method using the Galerkin procedure. Compute the coefficient matrix and the right hand vector of the 2-term approximation. Algebraic polynomials may be used as the approximating functions. [14]

4) The following differential equation arises in connection with heat transfer in a rod of length L, which is perfectly insulated round the circumference:

$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+Q=0$ in 0<x<L, The left end of the boundary (x=0) has a constant temperature $T_0$ and the right end is exposed to atmosphere of temperature $T_a$, where convective heat transfer takes place.

Derive the weak form of the governing equation and using Galerkin's procedure, obtain the element equation. Using two linear elements, estimate steady state temperatures at x=L/2 and at x=L. Take L = 0.2m, Q = 20 W/m$^{-3}$, k = 10.0 Wm$^{-1}$°C$^{-1}$, h = 25 Wm$^{-2}$°C$^{-1}$, $T_0$ = 50°C, $T_a$ = 5°C. [14]

## Group B

5) Compute the transverse deflection and slope at the right end (node 3) of the beam as shown in Fig. 2. Use two beam elements of equal length. The right end is supported by a linear spring as shown. No derivation is necessary and symbols have usual meaning.

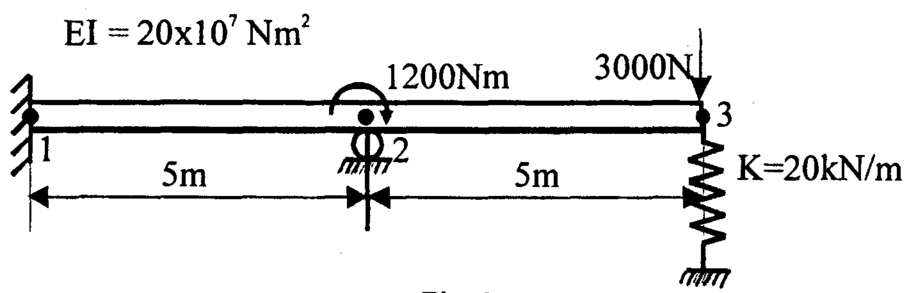



Fig. 2

The stiffness matrix of a typical beam element of length L and flexural rigidity EI may be taken

$$\text{as } [k_e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

[14]

6) Calculate the stiffness matrix and load vector of the element shown in Fig. 3. Thermal conductivity is 15W/m-°C and convection coefficient is 20 W/m$^2$-°C. Convection occurs at the edge i-m. The free-stream temperature is 15°C and there is a uniform heat generation 100W/m$^3$. The thickness may be taken as 1m and both the surfaces of the triangular element are insulated. Other two edges of the element are within the domain. Use appropriate form of the governing equation and boundary condition applicable to the case stated. Derivation of the weak form is to be done. [14]

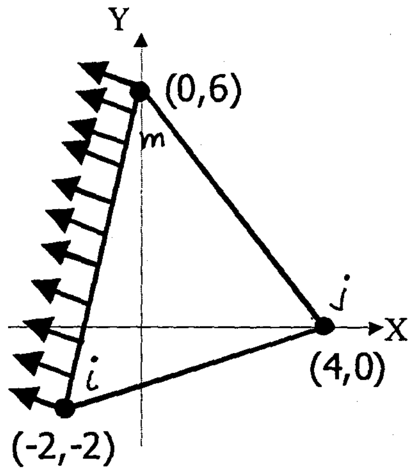



Fig. 3

7) For the 3-node plane stress element shown in Fig. 4, determine its element equation. The connectivity of the element is i-j-k. The element has a pressure of magnitude 2000 unit perpendicular to the side j-k and is subjected to a temperature rise of 30°. Take E = 30E6, υ = 0.25, t = 1.0, α = 7E-6. Symbols have their usual meaning and all the values are in consistent units. No derivation is necessary. [14]

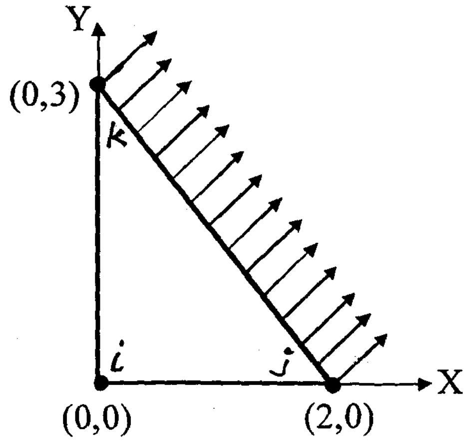



Fig. 4