B.E. (MECH.) Part III 6TH SEMESTER FINAL EXAMINATION, 2013

NUMERICAL METHODS IN ENGINEERING (ME 605)

Full Marks: 70 **Time: 3 hrs**

Questions of each Half must be answered in a separate answerscript

FIRST HALF

Answer Q. 5, and any TWO from the rest

1a) Apply Gaussian Elimination Method with partial pivoting to solve the following equation set.

$$x_1 + x_2 + x_3 = 9$$
$$2x_1 - 3x_2 + 4x_3 = 13$$
$$3x_1 + 4x_2 + 5x_3 = 40$$

b) What is an ill-conditioned equation set? Calculate the condition index of the above equation set and comment on its condition. [8+3=11]

2) The following equation set is to be solved by the iterative Gauss-Seidel method.

$$x_1 + 4x_2 - x_3 = -5$$
$$x_1 + x_2 - 6x_3 = -12$$
$$3x_1 - x_2 - x_3 = 4$$

Will convergence of the above equation set depend on the choice of initial guess? State your answer with reason (no derivation is necessary). Choose appropriate solution strategy for iterative solution of the equation set. Show three iterations of the stated method with the initial guess as: $x_1 = x_2 = x_3 = 0.0$. [11]

3a) A set of x-y data is tabulated below. The data is to be fit by a curve having the form $y = c_1 + c_2 x^2$ in the least square sense. Use matrix formulation of the least square procedure giving equal weights to all the data. Solve the resulting equation set to obtain the unknown coefficients c_1 and c_2. No derivation is necessary.

x	1	2	2.5	3.5	4.0
y	3.8	10.0	15.0	26.0	33.0

b) A set of n number x-y data is to be fit by an appropriate polynomial. If the relationship between x and y is not known, how can you decide on the degree of the approximating polynomial? [8+3=11]

4a) State the general steps of the Runge-Kutta method of order n in solving an initial value problem given by $\frac{dy}{dx} = f(x, y)$, subject to $y=y_0$ at $x=x_0$, with a step size h. Hence show that the Midpoint method belongs to the Runge-Kutta family of order 2.

b) Estimate y at x=1.2 using the Midpoint method for the initial value problem given by $\frac{dy}{dx} = yx^2 - 1.2y$, subject to y=1.0 at x=0. Take the step size as 0.1. [6+5=11]

5. Determine y at x=1.2 and at x=1.4 for the ODE given as $\frac{dy}{dx} = x^2 + y^2$, the initial condition being y=1.5 at x=1.0. Take step size as 0.2. Employ self starting type modified Euler's method with two corrections for the first step and classical fourth order Runge-Kutta method for the second step. [13]

SECOND HALF

Answer Q6 or Q7; Q8 or Q9; and Q10 and Q11

6. (a) Describe and give the graphical representation of the bisection method of solving algebraic and transcendental equation.

(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by bisection method.

7. (a) Describe and give the graphical representation of the Newton-Raphson method of solving algebraic and transcendental equation.

(b) Using Newton-Raphson method, find a real root, correct to 3 decimal places, of the equation $\sin x = {}^{x}/_{2}$ given that the root lies between ${}^{\pi}/_{2}$ and π.

8. (a) Outline the Graeffe's root-squaring method of solving algebraic and transcendental equation.

(b) Using Graeffe's method, find the real roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$.

9. (a) Describe Lin-Bairstow's method to determine a quadratic factor of a polynomial.

(b) Find a quadratic factor of the polynomial $f(x) = x^3 - x - 1$ by Lin-Bairstow's method.

10. Compute $I = \int_0^1 \frac{dx}{1+x^2}$ using the trapezoidal rule with h =0.5, 0.25 and 0.125. Then obtain a better estimate using Romberg's method. Compare your results with the true value.

11. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1$, and $u(0,t) = u(1,t) = 0$. Use Bender-Schmidt's and Crank-Nicolson formulae to compute the value of $u(0.6, 0.04)$ and compare the results with the exact value.