BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

B.E. (Mechanical) 6th Semester Final Examination, 2012

NUMERICAL METHODS IN ENGINEERING (ME- 605)

ANSWER SIX QUESTIONS TAKING THREE FROM EACH HALF

Full Marks: 70 **Time: 3 Hours**

FIRST HALF

1a. Solve the following equation set by the LU-Decomposition method. Employ partial pivoting.

$$-x_1 + x_2 - x_3 = -2$$
$$-2x_1 + x_2 + 2x_3 = 6$$
$$x_1 + x_2 + x_3 = 6$$

b. Write the pseudocode of the forward elimination step (without partial pivoting) of the LU-Decomposition method and obtain an expression of the number of the multiplication/division FLOPS required in this step (in terms of number of equations n).

2a. Derive the convergence criteria of the Gauss-Seidel method in solving a set of n linear algebraic equations. Is it a necessary or a sufficient condition?

b. The following equation set is to be solved by the iterative Gauss-Seidel method. Determine the values of **x** after two iterations. Relaxation parameter and initial guess may be taken as 0.8 and $\{x\} = [10 \quad 10 \quad 10]^T$ respectively.

$$\begin{bmatrix} -2 & -1 & 10 \\ -2 & 10 & -2 \\ 10 & -2 & -3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 120 \\ 154 \\ 205 \end{Bmatrix}$$

3. For the dataset given below obtain the best fit in the least square sense giving equal weights to the errors in y. The approximating function to be used for the fit may be taken as $y = k_1 x e^{-k_2 x}$. If you use any formula, prove it.

x	1	1.5	2	4	6	8
y	1.2	1.4	1.5	1.1	0.6	0.3

4a. How cubic splines are constructed numerically from a set of given data? Explain briefly the steps with reference to the natural cubic spline. Derivation is not necessary.

b. Estimate y at x=0.2 for the initial value problem $\frac{dy}{dx} = y^2 + x$, subject to y=1.0 at x=0; with a step size h=0.1. Employ self-starting type predictor-corrector method for the first step and non self-starting type predictor-corrector method for the second step. For both the steps, exercise single correction.

5a. Write a pseudocode to implement a single step calculation in estimating y for the initial value problem given in 5(b) using the Mid-point method.

b. Estimate y at x=0.2 by the classical 4th order Runge-Kutta method for the initial value problem given by $\frac{dy}{dx} = y + x$, subject to y=1.0 at x=0. Step size h=0.1.

6.a) Find a real root of the equation $\sin x = 1 - x$ within the interval $(0,1)$ using Newton-Raphson method, correct to three decimal places.

b) Find a root of the equation $x^3 + 2x - 5 = 0$ between 1 and 2 using the method of bisection correct to two decimal places.

c) Show that the error introduced in Trapezoidal rule is proportional to the square of the length of each sub-intervals.

7.a) Solve the following system of non-linear equations applying Newton-Raphson method, correct to three decimal places.

$$x^2 + y = 11 \qquad y^2 + x = 7$$

b) Using Lagrangian interpolation formula on the following tabulated values, find the value of y at $x = 26$.

x	14	17	31	35
y	68.7	64.0	44.0	39.1

8.a) Compute the following integral with 10 sub-intervals using both Trapezoidal and Simpson's 1/3rd rule. Compare both the results with actual solution.

$$I = \int_0^1 \cos x \, dx$$

b) In the following table, values of y for consecutive values of x are given. Find the polynomial which approximates these values using Newton's forward difference interpolation formula. Also find the value of y for $x = 2$.

x	3	4	5	6	7
y	13	21	31	43	57

9.a) Derive the Gauss-quadrature relation when $n = 2$ and apply that to evaluate the following integral. Compare the solution with analytical result.

$$I = \int_1^2 \frac{dx}{1+x}$$

b) Explain the procedure of finding roots of a polynomial of n^{th} order following Bairstow's method.

10.a) Solve the following boundary-value problem with $h = 0.5$ and $h = 0.25$ over the interval $[0,1]$ and find the value of y at the mid-point of the interval for both cases.

$$y'' - 64y + 10 = 0$$
$$y(0) = y(1) = 0$$

b) The following set of tabulated values are given for x and y. Find the values of first and second derivatives at $x = 1$.

x	0	1	2	3	4	5	6
y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309