B.E.(All Branches) 2nd Semester Final Examination , April 2012

Mathematics-II (MA-201)

Time : 3 hours | Full Marks : 70

(Use separate answerscript for each half)

FIRST HALF

(Answer Question No.1 and any TWO from the rest.)

1. Answer any **three** questions. (3x5)

(a) Reduce the following matrix to its normal form and hence find its rank in the two cases when

(i) a=-1 and (ii) $a \neq -1$:

$$A = \begin{bmatrix} a & -1 & -1 \\ -1 & a & -1 \\ -1 & -1 & a \\ 1 & 1 & 1 \end{bmatrix}.$$

(b) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Write down the corresponding matrix equation satisfied by A . Hence find A^{-1} and A^9 .

(c) State and prove the First Mean-Value Theorem of Integral Calculus.

(d) Define vector space. Prove that if W_1 and W_2 be any two sub-spaces of a vector space V over the field F, then $W_1 \cap W_2$ is also a sub-space of V over F. Give an example in support of the theorem.

2. (a)Apply First Mean Value Theorem of Integral Calculus to prove that if f(x) (> 0) is continuous in [a,b] then

$$\int_a^b \{f(x)\}^2 dx = (b-a)\, f(c)f(d), \text{ where } c, d \in [a,b].$$

(b) (i) Define eigen value and eigen vector of a square matrix.

(ii) If λ ($\neq 0$) is an eigen value of a non-singular matrix A , show that λ^{-1} is an eigenvalue of A^{-1} .

(iii) If λ_1 , λ_2 , λ_3 be the eigen values of the matrix A = $\left[a_{ij}\right]_{3x3}$, show that λ_1^2 , λ_2^2 , λ_3^2 are the eigen values of the matrix A^2 .

(iv) Prove that the matrices A and $P^{-1}AP$ have the same eigen values.

(v) Prove that the eigen vectors corresponding to two distinct eigen values of a real symmetric matrix are orthogonal.

[3+(1+1+1+2+2)]

3. (a)Test for convergence the following improper integrals:

(i) $\int_0^{\frac{\pi}{2}} \frac{cosx}{x^n}$ dx , (ii) $\int_1^{\infty} e^{-x} \frac{sinx}{x^2}$ dx .

(b) Investigate for which values of λ , μ the system of equations

x + y + z = 7 , x + 2y +3z = 10 , x + 2y + λz = μ

have (i) no solution , (ii) unique solution, (iii) an infinite number of solutions.

[(3+2)+5]

4. (a) Prove that

$$2^{2m-1}\Gamma(m)\Gamma(m+\tfrac{1}{2}) = \sqrt{\pi}\,\Gamma(2m), m > 0 .$$

OR

Prove that

$$\int_0^{\infty} e^{-x^4}\, dx \times \int_0^{\infty} e^{-x^4} x^2\, dx = \frac{\pi}{8\sqrt{2}} .$$

(b) Show that the vectors (1,2,1), (2,1,0) , (1,-1,2) form a basis of the vector space $V_3(F) = \{(x, y, z): x, y, z \epsilon F\}$ over the field F of real numbers.

(c) If A be a skew-symmetric matrix and (I+ A) be a non-singular matrix, then show that

B = (I– A) $(I + A)^{-1}$ is orthogonal.

[4+3+3]

SECOND HALF

Answer **question no. 5 and any two from the rest**

5. **Answer any three : (3 x 5 = 15)**

a) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$.

b) Determine the particular integral of $\frac{d^2y}{dx^2} + 4y = x\cos x$.

c) Solve $\frac{d^2y}{dx^2} = xe^x$.

d) If $f(t)$ is a periodic function of period a, then show that $L\{f(t)\} = \frac{1}{1-e^{-ap}}\int_0^a f(t)e^{-pt}dt$,

where $L\{f(t)\}$ is the Laplace transform of $f(t)$ with parameter p.

e) Determine the Laplace transform of $f(t) = e^{3t}\sin^2 5t$.

6. a) Solve the differential equation $x^2\frac{d^2y}{dx^2} - 2y = \frac{1}{x}$.

b) An electric circuit consists of an inductance L , capacitance C and an e.m.f E. Find the charge q and the current i when $E = E_0 \cos wt$ and the initial conditions are

$q = q_0$ *and* $i = i_0$ *at* $t = 0$; i, q satisfying the equations

$$\frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{E_0}{L}\cos wt,\ i = \frac{dq}{dt}.$$ **(5+5)**

7. a) Determine the series solution of the differential equation

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ in the neighborhood of $x = 0$.

b) Determine the particular integral of $\frac{d^2y}{dx^2} - 4y = e^{2x}$. **(7+3)**

8. a) Show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$, $when\ n \neq m$, where $P_l(x)$ is the Legendre polynomial of degree l.

b) Determine the Laplace transform of $f(t)$, where

$$f(t) = \frac{t}{T}, \; when\ 0<t<T$$
$$= 1 \;, \; when\ t>T$$

(5+5)

9. a) Determine the Laplace inverse of $\frac{1}{p^2(p^2-4)}$ by using convolution theorem, where p is parameter of Laplace transform.

b) If $L\{f(t)\} = \bar{f}(p)$, then prove that $L\{f(t-a)H(t-a)\} = e^{-ap}\bar{f}(p)$, where $\bar{f}(p)$ is the Laplace transform of $f(t)$ with parameter p and $H(t)$ is the Heaviside function defined by

$$H(t-a) = 1 \quad for\ t>a$$
$$= 0 \quad for\ t<a$$

Using the above result compute inverse Laplace transform of $\frac{pe^{-2p}}{p^2+1}$.

(5+5)