Bengal Engineering and Science University, Shibpur

B. E 1st Semester Final Examination, 2012

Mathematics-I

(MA-101)

Time: 3 hours **Full Marks: 70**

Use separate Answer Script for each Half.

First half

Answer question no.1 and any two from the rest

1. **Answer any three(3x5=15)**

a) State and prove Leibnitz's theorem.

b) In the M.V. Theorem,

$f(x+h)= f(x)+hf'(x+\theta h)$, $0<\theta<1$

show that the limiting value of θ is ½ as $h \to 0$ when $f(x) = \cos(x)$.

c) Prove that the asymptotes of the cubic $(x^2-y^2)y - 2ay^2+5x = 7$ form a triangle of area a^2.

d) Verify Euler's theorem for the function

$U(x , y) = \cos(\frac{x^2-y^2}{2xy})$

e) Examine for extreme values of the function $x^3+ y^3 - 3axy$.

2. (a) State and prove the Euler's theorem on homogeneous function of two variables.

(b) If $x^2+y^2+z^2 - 2xyz = 1$, show that

$$\frac{dx}{\sqrt{1-x^2}}+\frac{dy}{\sqrt{1-y^2}}+\frac{dz}{\sqrt{1-z^2}} = 0$$

((1+3)+6)

3. (a) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $Y^2 = 4ax$, then show that $\rho_1^{-2/3} + \rho_2^{-2/3} = (2a)^{-2/3}$.

(b) Using Lagrange's multiplier method show that the largest rectangle with a given perimeter is a square.

(5+5)

4. (a) Expand the function sin(x) in a finite series in power of x with the remainder in Lagrange's form.

(b) If f(x) = tan(x) then prove that

$f^n(0) - {}^nC_2 f^{n-2}(0) + {}^nC_4 f^{n-4}(0) - \ldots = \sin(n\pi/2)$. (5+5)

Second half

Answer any **THREE** questions from this half. The questions are of equal marks. 2 Marks are reserved for general proficiency in this half.

5. (a) Show that a necessary condition for convergence of an infinite series $\sum u_n$ is that $\lim(u_n) = 0$ as $n \to \infty$. Show through an example that it is not sufficient condition of convergence.

(b) Show that the series $\sum \frac{3.6.9......3n}{7.10.13.....(3n+4)} x^n$, x>0 converges for $x \le 1$, and diverges for $x > 1$

6+5=11

6. (a) Show that the positive term series $\sum \frac{1}{n^p}$ is convergent if and only if p > 1.

(b) Test the convergence of the following series

i) $\sum \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{1.5}}$ ii) $\sum_{n=1}^{\infty} \frac{1}{n^2 \log n}$

7+4=11

7. (a) Find the maximum value of the directional derivative of $f(x,y,z)=x^2+z^2-y^2$ at the point (1, 1, 2). Also find the direction in which it occurs.

(b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at (2, -1, 2)

(c) Find the value of the constant p so that the vectors $(2\vec{i}-\vec{j}+\vec{k})$, $(\vec{i}+2\vec{j}-3\vec{k})$ and $(3\vec{i}+p\vec{j}+5\vec{k})$ are coplanar.

5+3+3=11

8. (a) State Gauss divergence theorem and Stoke's theorem. Evaluate $\oint_C \vec{F}.d\vec{r}$ where $\vec{F}=-y^2\vec{i}+yx\vec{j}$ and C is the square cut in the first quadrant by the line x=1 , y=1.

(b) Evaluate $\oiint_S \vec{F}.\vec{n}ds$ where $\vec{F}=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$ and S is the surface of the rectangular parallelepiped $0\le x\le a$, $0\le y\le b$, $0\le z\le c$.

7+6=11

9. (a) Prove that div(Curl $\vec{F}$) =0. Find the $grad(r^3\vec{r})$ where $\vec{r}$ denotes the position vector of the point P(x,y,z) and $r=|\vec{r}|$.

(b) If the vectors $\vec{A}$ and $\vec{B}$ are irrotational, then show that $\vec{A}\times\vec{B}$ is solenoidal.

(c) Evaluate $\iint_D (x^3y)dxdy$ where D is the region enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant.

4+3+4=11

10. (a) Find the centroid and moments of inertia relative to x-axis and y-axis of the plan region R having mass density $(x+y)$ and bounded by the parabola $x=y-y^2$ and the straight line $x+y=0$.

(b) Calculate the volume of a solid whose base is in xy plane and is bounded by the parabola $y=4-x^2$ and the straight line $y=3x$, while the top of the solid is the plane $z=x+4$.

6+5=11