Ex/BBSUS/IT-404/10

B.E. (IT) Part-II 4th Semester Examination, 2010

Formal Language and Automata Theory

(IT-404)

Time: 3 Hours **Full Marks: 70**

Q 1 is compulsory. Answer any five from the rest.

Ql. Answer any ten. **10 X 2 = 20**

a) If w is any string over an alphabet {0, 1}, then w* denotes the reverse of w. If L is any language, then L^R denotes the language consisting of the reverses of all strings in L. Then If L is any language, then the language LL^R must be equal to {ww* | w belongs to L}. Is it true or false? Explain with reasons.

b) If L is a recursive language and R is a recursively enumerable language then L - R must be recursively enumerable. Is it true or false? Explain with reasons.

c) Is the r.e. languages closed under both union and intersection? Explain with reasons.

d) Consider three decision problems PI, P2 and P3. It is known that PI is decidable and P2 is un-decidable. Which one of the following is true? Explain with reasons.
(A) P3 is decidable if PI is reducible to P3.
(B) P3 un-decidable if P3 is reducible to P2.
(C) P3 is un-decidable if P2 is reducible to P3.
(D) P3 is decidable if P3 is reducible to P2's complement.

e) Let £ = {0}. Let L = {0^j where j is in Geometric Progression} and R - {0^k where k is in arithmetic progression. The language L H R is regular. Is it true or false? Explain with reasons.

f) Let LI, L2 and L3 be three languages over an alphabet I. Let LI U L2 U 13 = E* and there's nothing common between LI, L2 and L3.

Then:

A) If all languages are recursive then all are r.e
B) If all languages are r.e. then they are recursive
C) Both A and B is true
D) Neither A nor B is true

g) Consider the following grammar G:

S~-->bS|aA|b

A—>bA|aB

B—>bB|aS|a

Let Na(w) and Nb(w) denote the number of a's and b's in a string w respectively. Find out the relation between Na(w) and Nb(w).

h) If LI is a r.e. language and L2 is recursive, then LI concatenated with L2 is r.e. language. Is it true or false? Explain with reasons.

i) If f is a function from the set A -> B. The number of different functions is 31. Find the cardinalities of A and B.

j) Let S = {0,1,2}. Find out the number of possible binary relations out of this member of the set.

k) Show that any co-finite set (complement of a finite set) is always regular.

1) Is DPDA and NDPDA equivalent? Explain with reasons.

Q2.

a) Show by construction that **(0*01 +10)*0* = (0 + 01 + 10)***

b) Show that the language { 0^ where p is a prime number} is not regular.

7 + 3 = 10

Q3. 5 + 5 = 10

Find out the regular expression accepted by the DFA below:

a) By GNFA method
b) By Arden's theorem

Q4.

a) Consider the following grammar:

S^0A0|1B1|BB
A -» C
B->S|A
C -»S|e

Remove null and unit production and convert it into CNF. Then apply CYK algorithm to show that 1001 belongs to the language produced by this grammar.

6 + 4 - 1 0

Q5.

a) Show that if L is a CFL so is L*.
b) If LI and L2 are two CFL's then is LI - L2 CFL?
c) If L is a language accepted by a PDA with a finite stack then is L regular? Explain with reasons.

4 + 3 + 3 = 10

Q6.

a) Show that the language L<j is the set of strings w such that the TM M whose code is w does not accept when given w as input, is non recursively-enumerable.
b) Show that the Universal language is recursively enumerable but not recursive.

5 + 5 = 10

Ql.

a) Design a Turing Machine to accept the language **{0" 1" 2"** |n >— **1}**
b) Consider a Turing Machine with the states {qO, ql, q2, qf} and the input symbol is {0,1}. There is only one extra tape symbol that is B. qO is the initial state and qf is the accepting state. The transitions are as follows:
5(q0, 0) = (ql, 1, R); 5(ql, 1) = (qO, 0, R); S(ql, B) = (qf, B, R)

Show that it accepts **(01)*0**

5 + 5 = 10

Q8.

a) Show that if a mod b = b mod a where a, b are positive integers, then a = b.
b) Show that any given 10 distinct positive numbers less than 100, that two completely different subsets sum to the same quantity. Use pigeon hole principle.

5 + 5 = 10

Q9.

a) Show that a Turing Machine halting problem is un-decidable.
b) Construct a DFA which shall accept LI intersect L2 where Ml and M2 are two DFA*s accepting LI and L2 respectively. 5 + 5 = 10