

B.E. IT Part II 4th Semester Examination, 2010

Sub: Signals and Systems (IT- 403)

Answer **SIX** questions taking **THREE** from each half. Two marks are reserved for neatness and to the point answer in each group.

Full Marks: 70

Time- 3 hours

1st Half

1. What is meant by energy and power signals? " A signal cannot be both energy and a power signal"- Justify. Prove that two complex functions $X_1(t)$ and $X_2(t)$ are orthogonal over an interval $(t_1 < t < t_2)$ if

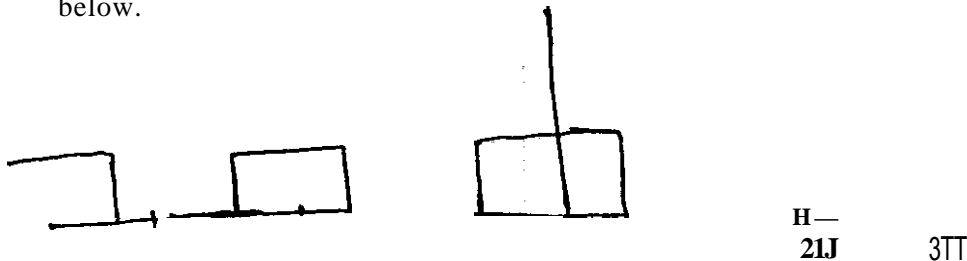
$$\int_{t_1}^{t_2} X_1(t)X_2^*(t)dt = 0 \quad \text{or}$$

where $X_2^*(t)$ and $X_1^*(t)$ indicate complex conjugation of $X_2(t)$ & $X_1(t)$, respectively.

What is the critical difference in the spectrum obtained from the trigonometric Fourier series and exponential Fourier series of; a periodic signal.

(3+2+4+2)

2. a) State and write down the mathematical form of Parseval's theorem. Write down the basic conditions for the existence of the Fourier series,
 b) Find the exponential Fourier series for the periodic square wave shown in Fig. 1 below.



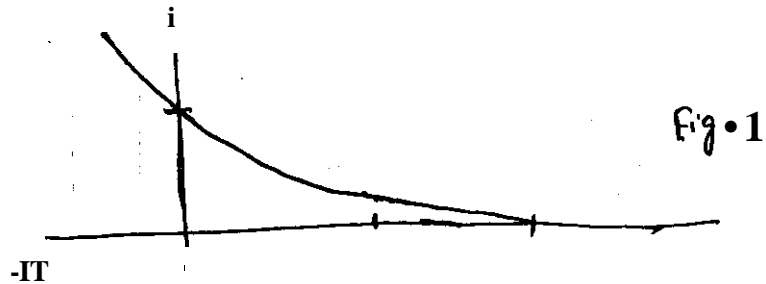
- c) Energies of the two energy signals $x(t)$ and $y(t)$ are E_x and E_y , respectively.
 i) If $x(t)$ & $y(t)$ are orthogonal, then show that the energy of the signal $x(t)+y(t)$ is identical to the energy of the signal $x(t)-y(t)$, and is given by $E_x + E_y$.
 ii) We define E_{xy} , the cross energy of the two energy signals $x(t)$ & $y(t)$, as

$$E_{xy} = \int_{-\infty}^{\infty} x(t)y^*(t)dt$$

If $z(t)=x(t)\pm y(t)$, then show that $E_z = E_x \pm E_y \pm (E_{xy} + E_{yx})$

(2+2)+4+>

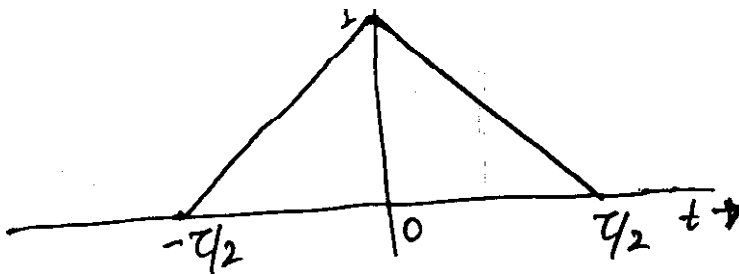
3. a) Find the compact trigonometric Fourier series over the interval $0 < t < n$ for the exponential function shown in Fig. 2. Show the periodicity of trigonometric Fourier series.



- b) Find the trigonometric Fourier series and sketch the corresponding spectra for the periodic impulse train shown in Fig. 3.

" =TS -TV : y° Tt' X|t, (6*5)

4. a) Write down the time differentiation property of Fourier transform and using the time differentiation property, find the Fourier transform of the triangle pulse $A(t/x)$ shown in



- b) Find the Fourier transform of the function $g(t) = \text{Sgnt}$.
 c) Comment on frequency domain description of Fourier transform for non-stationary signal. Can you suggest the name of other tools to overcome this type of problem?

(2+4-1-3+2)

5.a) Write down the conditions for distortionless transmission of signals through a linear system. What is meant by linear phase filter? How does it differ from linear filter?

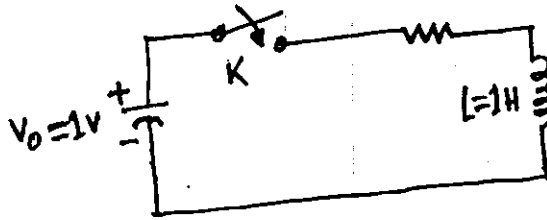
b) A bandpass signal $g(t)$ of bandwidth $\Delta\omega = 2000$ rad/sec centered at $\omega = 10^5$ rad/sec is passed through the RC filter with $R = 10^3 \Omega$ and $C = 10^{-6}$ F. If over the passband, the variation of less than 2% in amplitude response and less than 1% in time delay is considered distortionless transmission, would $g(t)$ be transmitted without distortion? Find the approximate expression for the output signal.

2nd Half

- 6.a) What are the advantages offered by sampling process? State and prove the Nyquist sampling theorem for a signal band-limited in W . What is meant by aliasing effect?
 b) Estimate the essential bandwidth W rad/sec for the signal $e^{-t} u(t)$ if the essential bandwidth is required to contain 95% of the signal energy.

(2+4+1)+4

7. a) Find the Laplace transform of $f(t) \wedge u(t)$, unit step function. Derive the time domain expression of current $i(t)$ for the RLC circuit with the capacitor initially charged to voltage V_0 as indicated in Fig. 5.



Write down the frequency shift property of Laplace transform.

- b) Find the inverse Laplace transform of

$$s + 1$$

$$s + 2s$$

- c) Write down the expression of initial and final value of $f(t)$ from $F(s)$. Calculate $i_j(0)$ when

$$4s + 6$$

$$' '' (25 + 1)(s + 2)$$

(1+3+1)+2+(3+1)

8. a) A binary-symmetric channel (BSC) error probability is P_e . The probability of transmitting '1' is Q , and that of transmitting 0 is $1-Q$. Determine the probabilities of receiving 1 and 0 at the receiver.
 b) An analog signal $x(t)$ of amplitude range $(-m_p, m_p)$, bandlimited to B Hz is sampled at a rate of $2B$ samples per second and is quantized to L^* uniform levels. Calculate mean square of the quantization error.
 c) Show that $|p_{,j} - 1| >$ where is the correlation coefficient of random variables x and y .
 d) Given $X \wedge \cos t$ and $Y = \sin t$, where t is an RV uniformly distributed in the range $(0, 2\pi)$, show that X and Y are uncorrelated but are not independent.

(2+ 4+2+3)

9. a) A low-pass filter transfer function $H(\omega)$ is given by

$$H(\omega) = \frac{1}{1 + jkcs \omega} \quad \text{---} \quad \omega^0 \wedge \wedge$$

An arbitrary pulse $g(t)$ band-limited to B Hz is applied at the input of this filter. Find the output $y(t)$.

b) Develop the expression of transfer function $H(\omega)$ that represents distortion caused by multipath effects.

c) How can Rayleigh density function be derived from two independent Gaussian RVs? Give an example of noise in electrical communication system that follows Rayleigh density.

d) A noise signal $n_i(t)$ with PSD $S_{n_i}(\omega) = K$ is applied at the input to an ideal differentiator. Determine the PSD and the power of the output noise signal $n_o(t)$.

(3+2+4+2)

10. a) What is meant by stationary random process? How does it differ from wide sense stationarity? Show that the random process $X(t) = A \cos(\omega_c t + \theta)$, where θ is an RV uniformly distributed in the range $(0, 2\pi/r)$, is a wide-sense stationary process.

b) Write down the properties of jointly Gaussian random variables. Prove that the response of a linear system to a Gaussian process is also a Gaussian process.

(2+1+3)+2+3