

DISCRETE MATHEMATICS (IT 303)

Full Marks: 35

Time: 2 hours

(Answer any five questions)

1. a) What is Proposition? 1+3+3=7
- b) Construct the truth tables for the following statements:
- i) $(p \rightarrow p) \vee (p \rightarrow \bar{p})$
- ii) $(p \vee \bar{q}) \rightarrow \bar{p}$
2. a) Evaluate the sum using generating function: 4+3=7
- $$1^2 + 2^2 + 3^2 + \dots + r^2$$
- b) Determine the discrete numeric function corresponding to the following generating function:
- $$A(z) = \frac{1}{1-4z^2}$$
3. a) Solve the following recurrence relation: 5+2=7
- $$a_r - 7a_{r-1} + 10a_{r-2} = 3^r, \text{ given that } a_0 = 0 \text{ and } a_1 = 3.$$
- b) What is equivalence relation?
4. a) Let ρ be an equivalence relation on a set S and $a, b \in S$. Then prove that $cl(a) = cl(b)$ if and only if $a \rho b$. 2+5=7.
- b) Let S be the set of all positive divisors of 30. Verify whether the binary relation \leq on S is Poset, where $a \leq b$ means "a is a divisor of b", for a, b in S . Draw the covering diagram of the given relation if the relation is Poset.
5. a) Let (G, o) be a group. Prove that a non-empty subset H of G forms a subgroups of (G, o) if and only if $a \in H, b \in H \Rightarrow a o b^{-1} \in H$. 4+3=7.
- b) If (G, o) be a group in which $(a o b)^2 = a^2 o b^2$ for all $a, b \in G$, prove that the group is abelian.
6. a) Show that the following mapping f is injective but not surjective
- $$f : \mathbb{Z} \rightarrow \mathbb{Q} \text{ defined by } f(x) = 2^x, x \in \mathbb{Z}. \quad \text{3+4=7.}$$
- b) Show that the following mapping f is a bijection. Determine f^{-1} .
- $$f : \mathbb{R} \rightarrow \mathbb{R} \text{ defined by } f(x) = 2x + 3, x \in \mathbb{R}.$$