B.E. IT Part II 3rd Semester Examination, 2012

Sub: Signals, Systems and Circuits (IT- 304)

Answer **SIX** questions taking **THREE** from each half. Two marks are reserved for neatness and to the point answer in each group.

Full Marks: 70 Time- 3 hours

1st Half

1. a) Define mathematically what is meant by signal power. Write the sampling property of the unit impulse function. Show mathematically that the energy of the sum of orthogonal signals is the sum of their energies.

b) Show that if $w_1=w_2$, the power of $g(t)=c_1 \cos(w_1 t+\theta_1) + c_2 \cos(w_2 t+\theta_2)$ is $[C_1^2+C_2^2+2C_1 C_2 \cos(\theta_1 - \theta_2)]/2$, which is not equal to $(C_1^2+C_2^2)/2$.

c) What is the critical difference in the spectrum obtained from the trigonometric Fourier series and exponential Fourier series of a periodic signal.

(2+2+2)+ 3+2

2. a) Show that the power of a signal g(t) given by

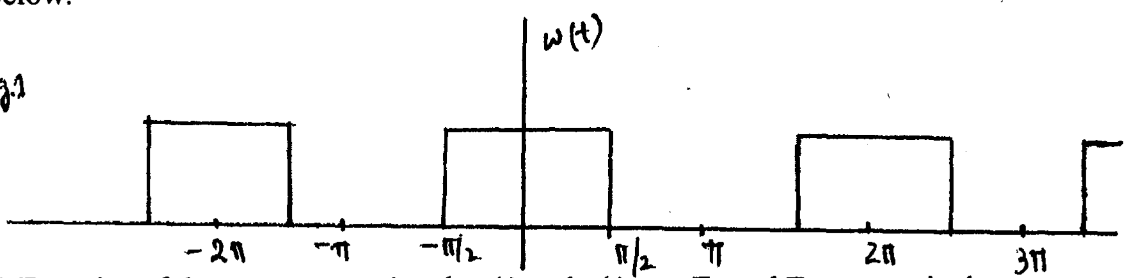
$$g(t) = \sum_{k=m}^{n} D_k e^{j w_k t} \quad , w_i \neq w_k \text{ for all } i \neq k$$

is (Parseval's theorem)

$$P_g = \sum_{k=m}^{n} |D_k|^2$$

b) Find the exponential Fourier series for the periodic square wave shown in Fig. 1 below.

Fig.1



c) Energies of the two energy signals x(t) and y(t) are E_x and E_y, respectively.

i) It x(t) & y(t) are orthogonal, then show that the energy of the signal x(t)+y(t) is identical to the energy of the signal x(t)-y(t), and is given by $E_x + E_y$.

ii) We define E_{xy}, the cross energy of the two energy signals x(t) & y(t), as

$$E_{xy} = \int_{-\infty}^{\infty} x(t)y^{*}(t)dt$$

If z(t)=x(t)±y(t), then show that $E_z = E_x + E_y \pm (E_{xy} + E_{yx})$

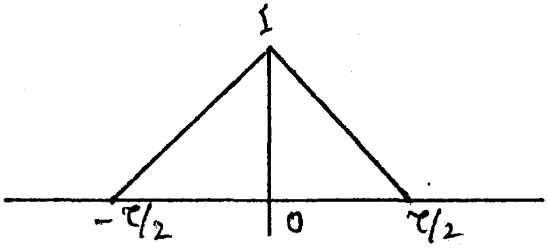
4+4+3

3. a) Show that Fourier transform of g(-t) is G(-w) and using this result find the Fourier transforms of $e^{at}u(-t)$ and $e^{-a|t|}$.

b) Write down Parseval's theorem for energy E_g of a signal g(t) and verify the same (Parseval's theorem) for the signal $g(t)=e^{-at}u(t)$ (a>0)

c) Write down the time differentiation property of Fourier transform and using the time differentiation property, find the Fourier transform of the triangle pulse $\Delta(t/\tau)$ shown in Fig. 2.

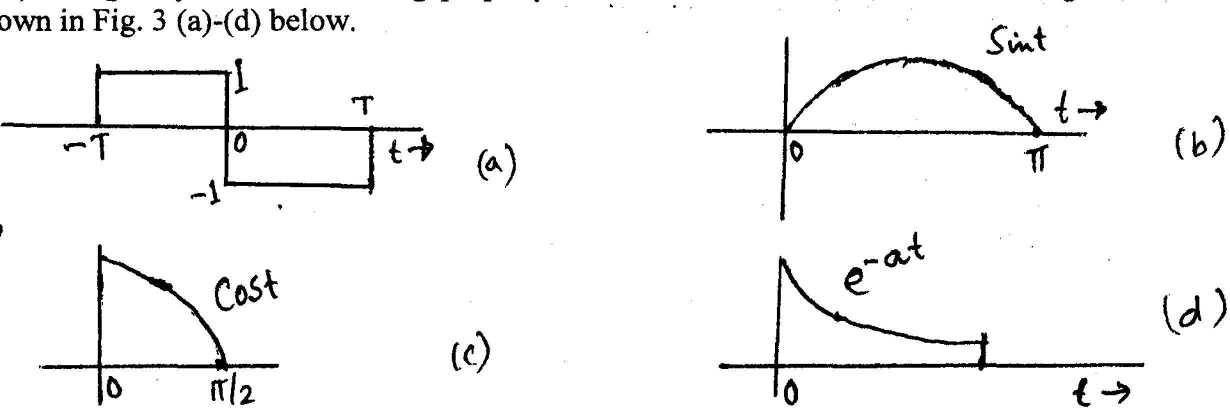
Fig. 2



(3+3+5)

4. a) Using only the time shifting property, find the Fourier transforms of the signals shown in Fig. 3 (a)-(d) below.

Fig. 3



b) Write down duality property of Fourier transform.
c) Comment on frequency domain description of Fourier transform for non-stationary signal. Can you suggest the name of other tools to overcome this type of problem?

(4x2+1+2)

5.a) Write down the conditions for distortionless transmission of signals through a linear system. What is meant by linear phase filter? How does it differ from linear filter?
b) A bandpass signal g(t) of bandwidth Δw= 2000 rad/sec centered at w= 10^5 rad/sec is passed through the RC filter with R= 10^3 Ω and C= 10^{-9} F. If over the passband, the variation of less than 2% in amplitude response and less than 1% in time delay is

considered distortionless transmission, would g(t) be transmitted without distortion? Find the approximate expression for the output signal.

5+6

2^{nd} Half

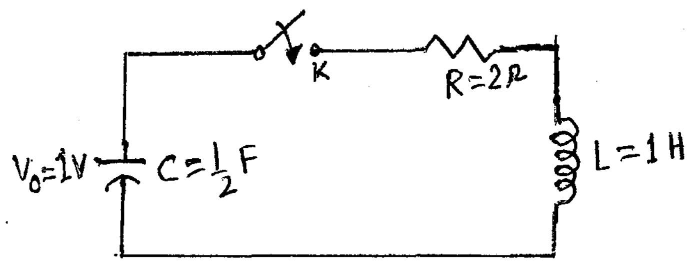
6.a) State and prove the Nyquist sampling theorem for a signal band-limited in W. What is meant by aliasing effect?

b) Estimate the essential bandwidth W rad/sec for the signal e^{-at} u(t) if the essential bandwidth is required to contain 95% of the signal energy.

(2+4+1)+4

7. a) Find the Laplace transform of f(t) $=e^{-at}$ u(t). Derive the time domain expression of current i(t) for the RLC circuit with the capacitor initially charged to voltage V_0 as indicated in Fig. 4.

Fig.4



Write down the frequency shift property of Laplace transform.

b) Find the inverse Laplace transform of

$$I(s) = \frac{7s-6}{s^2 - s - 6}$$

c) Write down the expression of initial and final value of f(t) from F(s). Calculate $i_i(0)$ and $i_i(\alpha)$ when

$$I_1 = \frac{10(2s+3)}{s(s^2+2s+5)}$$

(1+3+1)+2+(2+2)

8.a) An analog signal x(t) of amplitude range ($-m_p$, m_p), bandlimited to B Hz is sampled at a rate of 2B samples per second and is quantized to `L' uniform levels. Calculate mean square of the quantization error.

b) A certain PCM channel consists of n identical links in tendem. The pulses are detected at the end of each link and clean new pulses are transmitted over the next link. If p_e is the probability of error in detecting a pulse over any one link, show that P_E, the probability of error in detecting a pulse over the entire channel (over the n links in tendem), is

$$P_E \approx n\,p_e$$

d) Given X=cos θ and Y=sin θ, where θ is an RV uniformly distributed in the range (0, 2 π), show that X and Y are uncorrelated but are not independent.

(4+3+4)

9.a) Over a certain binary communication channel, the symbol 0 is transmitted with probability 0.4 and 1 is transmitted with probability 0.6. It is given that P(e/0)=10^{-6} and P(e/1)=10^{-4}, where p (e/x_i) is the probability of detecting the error given that x_i is transmitted. Determine P (e), the error probability of the channel.

b) A binary source is emitting bits 1 and 0 randomly; they are assigned by (i) a signal of amplitude A_p and - A_p, respectively (polar) and (ii) A_p and 0, respectively (ON-OFF), respectively. If the transmitted signal is contaminated by additive Gaussian noise of zero mean and variance σ_n, calculate the error probabilities of received data for both polar and on-off cases.

c) How can Rayleigh density function be derived from two independent Gaussian RVs? Give an example of noise in electrical communication system that follows Rayleigh density.

d) For a certain binary nonsymmetric channel it is given that $P_{y/x}(0/1)$=0.1 and $P_{y/x}(1/0)$=0.2, where x is the transmitted digit and y is the received digit. If $P_x(0)$=0.4, determine P_y (0).

(2+3+4+2)

10. a) What is meant by autocorrelation function of a random process? Show that the random process X(t)=A cos(w_ct+Θ), where Θ is an RV uniformly distributed in the range (0, 2 π), is a wide-sense stationary process.

b) Using the expression of covariance, show how a Gaussian random process is specified and write down the expression of same for stationary Gaussian process.

C) Prove that the response of a linear system to a Gaussian process is also a Gaussian process.

(2+3)+ 3+3