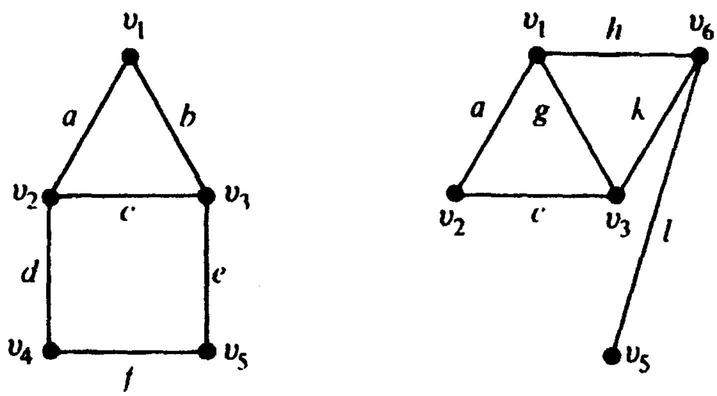
BE 3rd Semester Examination, 2012
Discrete Mathematics and Graph Theory
IT – 303
Information Technology Department

Full Marks: 70 **Time: 3 Hours**

Answer any five questions.

1. a) State the required properties of axioms. [3]
   b) What is known as partially quantified predicate? [2]
   c) State the principle of strong induction. [3]
   d) What is a total order? [2]
   e) Draw the Hasse diagram for the set, S={1,2,3,4,5} over the relation $\leq$ and show that it is a partial order. [4]

2. a) Prove that a simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges. [6]
   b) Find the ring sum of G_1 and G_2. [4]



G_1 G_2

   c) Define i) Euler graph ii) Hamiltonian circuit. [2+2]

3. a) Prove that in a n-vertex binary tree, there are (n+1)/2 leaf nodes. [3]
   b) Prove that a connected planar graph with n vertices and e edges has e - n + 2 regions. [4]
   c) Prove that $K_{3,3}$ is nonplanar. [4]
   d) What is a self-dual graph? Explain with an example. [3]

4. a) Prove that the vertices of any planar graph can be properly colored with five colors. [5]
   b) Define i) point covering number ii) line independence number. [4]
   c) What is a regular digraph? Explain with an example. [3]
   d) Explain what you understand by point critical graph. [2]

5. a) What is generalized pigeon-hole principle? If there are 45 persons in a room, then at least how many of them will share their month of birth? Explain. [2+3]
   b) Prove that (Z_n, X) is not a group. [operation X stands for multiplication] [4]
   c) Define commutative ring. Give an example of a commutative ring. [5]

6. a) Show that permutation with the operation composition (a permutation applied after another) is a non-abelian group. [4]
   b) What is a Galois Field? Explain the operations in GF(2). [4]
   c) Let the alphabet, Σ, contains a set of symbols, and Σ^* is the set of all strings containing the symbols in Σ. Give a recursive definition of Σ^*. [4]
   d) Give the recursive definition of the function f(n) = 2n+1, n=0,1,2, ... [2]