Bengal Engineering & Science University, Shibpur

B.E. Part IV (ETC), 7th Semester Final Examination, 2013-14

Digital Signal Processing (ET 704)

Full Marks: 70 **Time: 3 hours**

Use separate answer scripts for each half

FIRST HALF

Answer question no. **1** *and any* **two** *from the rest*

1. a) Consider the system illustrated in Figure 1. The output of a linear time invariant (LTI) system with an impulse response $h[n] = (1/4)^n u[n+10]$ is multiplied by a unit step function $u[n]$ to yield the output of the overall system. Answer each of the following and briefly justify your answer.

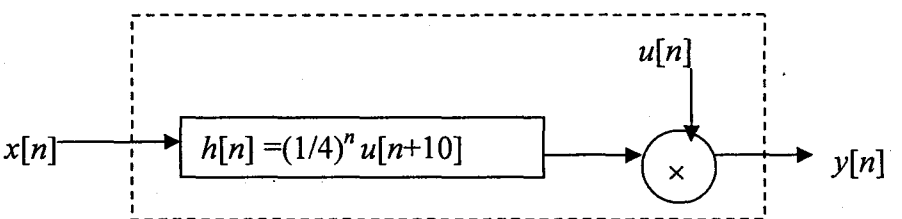

Figure 1

(i) Is the overall system LTI?
(ii) Is the overall system causal?

b) Is it feasible to plot a close replica of the magnitude spectrum of a length-4 discrete-time sequence using its 4-point DFT samples? Justify your answer

c) Determine whether each of the following signals is periodic. If the signal is periodic, state the period.

i) $x[n] = \exp(j3\pi n/4)$, (ii) $x[n] = [\sin(\pi n/4)]/(\pi n)$

5+6+4

2. a) Determine the discrete-time Fourier transform of the following sequence. Plot its magnitude spectrum

$$x[n] = a^n u(n), \quad |a|<1$$

b) If DFT of an N-length sequence $x[n]$ $(0 \le n \le N-1)$ is given by the sequence $X[k]$, then prove that

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\exp(j2\pi kn/N), \qquad 0 \le n \le N-1$$

5+5

3. a) An LTI discrete system is characterized by the following system function

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$

Specify the ROC of $H(z)$ and determine the impulse function for the following conditions

(i) the system is stable (ii) the system is causal

b) Prove that an LTI causal system is BIBO stable if and only if poles of its system function lie inside the unit circle.

5+5

4. a) Explain the method of determining linear convolution of two-discrete time finite-length sequences using DFT

b) Determine the DFT of the sequence $x[n] = \{1, 2, 4, -2\}$

5+5

5. Prove that minimum number of (complex) multiplication operations required to perform 8-point DFT of a causal sequence (of length 8) is 12. Show necessary mathematical steps and draw a neat signal flow graph.

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SECOND HALF

*Answer question no. **5** and any **three** from the rest*

6. Determine the system function of the filter shown in Fig. A.

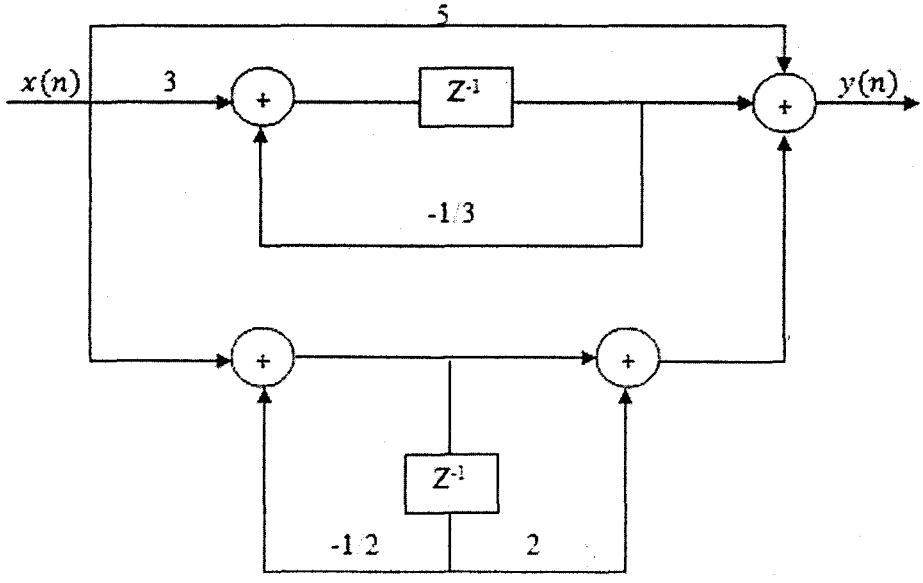

Fig. A

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7. Consider a causal LTI system with system function $H(z) = \frac{1-a^{-1}z^{-1}}{1-az^{-1}}$, where 'a' is real.

(a) Write the difference equation that relates the input and output of this system.
(b) For what range of values of 'a' is this system stable?
(c) For a=0.5, plot the pole-zero diagram and shade the ROC in z-plane.
(d) Find the impulse response for this system.
(e) Show that this system is an all-pass system. Also specify the value of the constant.

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8. Show that the impulse response $h(n)$ of an FIR filter of length $(2M+1)$ may be obtained from its frequency samples $H_k\ \forall k = 0,1,2,\ldots..M$ in accordance with the following equation:

$$h(n) = \frac{1}{2M+1}\left[H_0 + 2\sum_{k=1}^{M} H_k \cos\left\{\frac{2\pi k(n-M)}{2M+1}\right\}\right], 0 \le n \le 2M$$

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9. Design a Butterworth filter using Impulse Invariance Transformation for the following specifications:

$$0.8 \le \left|H(e^{j\omega})\right| \le 1 \quad for\ 0 \le \omega \le 0.2\pi$$
$$\left|H(e^{j\omega})\right| \le 0.2 \quad for\ 0.6\pi \le \omega \le \pi$$

Assume T=1 second.

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10. Design a single-pole low-pass digital filter with a 3-dB bandwidth of 0.2π. Hence realize the filter using hardware blocks.

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11. Realize the system described by the difference equation: $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ in cascade and parallel form.

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