B.E. Part IV (ETC), 7th Semester Final Examination, 2012-13

Digital Signal Processing (ET 704)

Full Marks: 70 **Time: 3 hours**

Use separate answer scripts for each half

FIRST HALF

Answer question no. 1 *and any* **two** *from the rest*

1. a) By direct evaluation of the convolution sum, determine the step response of a linear time-invariant system whose impulse response is

$$h[n] = a^{-n}u[-n] \qquad 0 < a < 1$$

b) Explain the justification of using DFT samples for plotting spectrum of a finite length discrete-time signal. Is it also justified to use DFT samples for plotting frequency response of an IIR filter?

c) Find the inverse Z-transform of the following.

$$X(z) = \frac{z^7 - 2}{1 - z^{-7}}, \qquad |z| > 1$$

4+5+6

2. a) Let $x[n]$ and $y[n]$ denote complex sequences and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective DTFTs. Show that

$$\sum_{-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$$

where $y^*[n]$ and $Y^*(e^{j\omega})$ represent complex conjugates of $y[n]$ and $Y(e^{j\omega})$ respectively. Hence, prove

$$\sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

b) Consider an LTI system with frequency response

$$H(e^{j\omega}) = \frac{1 - e^{-j2\omega}}{1 - e^{-j4\omega}}, \qquad -\pi < \omega \le \pi$$

Determine the output $y[n]$ for all n if the input $x[n]$ for all n is given as

$$x[n] = \sin(\pi n/4)$$

5+5

3. a) Determine the step response of the system described by the following relation.

$$y[n] = a\,y[n-1] + x[n]$$

where $x[n]$ and $y[n]$ represent input and output of the system respectively and 'a' is an arbitrary constant.

b) Prove that the **necessary and sufficient** condition for an LTI discrete-time system to be BIBO stable is its impulse response be absolutely summable.

5+5

4. a) Use circular convolution operation to determine the linear convolution of the following sequences.

$$x_1[n] = \{1, 2, 4\},\ x_2[n] = \{-2, 3, -1\}$$

b) Explain the method of determining linear convolution of two-discrete time finite-length sequences using DFT.

5+5

5. a) Determine power and phase spectrum of the following discrete-time signal

$$x[n] = \{-1, 3, 1, -2\}$$

b) Discuss the periodogram method of estimating power spectral density of a wide sense discrete time random process

5+5

SECOND HALF

*Answer question no. 6 and any **three** from the rest*

6.
(a) Impulse response of a system is given by $h(n) = a^n u(n)$. Formulate the relationship between input and output of the system by means of difference equation.
(b) What do you mean by finite word-length effect?
(c) Where are the zeros of minimum-phase FIR filters located?

2+2+1

7. Consider the system $y(n) = 0.5y(n-1) + x(n)$
(a) Compute its response for $0 \le n \le 5$ to the input $x(n) = 0.25^n u(n)$, assuming infinite-precision arithmetic.
(b) Compute the response of the system to the same input, assuming finite-precision sign-and-magnitude fractional arithmetic with five bits (i.e., the sign bit plus four fractional bits). The quantization is performed by truncation.
(c) Compare the results obtained in parts (a) and (b).

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8. Design a 5-tap FIR band-pass filter with a lower cut-off frequency of 1600 Hz, an upper cut-off frequency of 1800 Hz and a sampling rate of 8000 Hz using rectangular window function. Determine the transfer function and difference equation of the designed FIR system and compute its magnitude response at 0, 0.25π, 0.5π, 0.75π and π radians.

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9. Design a Butterworth filter using Impulse Invariance Transformation for the following specifications:

$$0.8 \le |H(e^{j\omega})| \le 1 \quad for\ 0 \le \omega \le 0.2\pi$$
$$|H(e^{j\omega})| \le 0.2 \quad for\ 0.6\pi \le \omega \le \pi$$

Assume T=1 second.

10

10. Convert the analog filter with the system function $H(s) = \frac{1}{(s+0.1)^2+16}$ into a digital IIR filter by means of Bilinear Transformation. The digital filter is to have a resonant frequency at 0.5π.

10

11. Obtain the cascade and parallel form realization of the system described by:

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2).$$

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