Bengal Engineering & Science University, Shibpur

B. E.(ETC) 3rd Semester Examination, 2012-13

Subject: Signals and Systems (ET 301)

Time: 3 hours **Full marks:70**

Use separate answer scripts for each half

FIRST HALF

(Answer Q.No.4 and any TWO from the rest)

1. (a) Write time reversal property of Fourier transform. Using time differentiation property, Find the Fourier transform of triangle pulse $\Delta(t/\tau)$.

   (b) How do you represent the Orthogonal signal in Space. Find the compact trigonometric Fourier series for the periodic signal $e^{-t/2}$. Write down Parseval's theorem.

   (c) Consider a causal LTI system with frequency $H(\omega)=\frac{1}{3+j\omega}$, for a particular input $x(t)$, this system is observed to produce the output $y(t)=e^{-3t}u(t)-e^{-4t}u(t)$, determine $x(t)$.

   [4+7+4]

2. (a) Compute the convolution of the following signals:
   $x_1(n)=x_2(n)=\delta(n+1)+\delta(n)+\delta(n-1)$.

   (b) Consider an LTI system with input output related the following equation:

   $y(t)=\int_{-\infty}^{t} e^{-(t-\tau)}x(\tau-2)d\tau$, What is the impulse response for this system?

   (c) Whether the system $y(t)=tx(t)$ is stable or not?

   (d) Write down the properties of auto correlation functions.

   [4+5+3+3]

3. (a) What do you mean by Distortion less Transmission? Describe the nature of distortion in audio and video signals

   (b) If $x(t)$ and $y(t)$ are the input and output of a simple RC low pass filter ($R=10^3\Omega$, $C=10^{-9}F$) determine the transfer function $H(\omega)$ and sketch magnitude, phase and time delay. For distortion less transmission through this filter, what is the requirement on the bandwidth of $x(t)$ if amplitude response variation within 2% and time delay variation within 5% are tolerable? What is the transmission delay? Find the output.

   (c) Write Sampling theorem. Find the Nyquist rate for the signal $x(t)=\frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$. What do you mean by Interpolation?

   [5+5+5]

4. Write short notes on (ANY ONE)

   (a) LTI system (b) Uniform and Nonuniform Quantization [5]

SECOND HALF
(Answer Q.No.5 and any TWO from the rest)

5. (a) Give an example of a pair of random variables which are uncorrelated but are not independent. Justify your example.

   (b) Explain the concept of pre-envelope and complex envelope of a bandpass signal.

   (c) Autocorrelation function $R_x(\tau)$ of any stationary process $X(t)$ is given by

   $$R_x(\tau) = \exp(-a|\tau|)$$

   The random process is applied at the input of a LTI system with impulse response $h(t) = 1/\pi t$. Determine power spectral density of the output random process.

   [3+6+6]

6. (a) A binary source generates digits 1 and 0 randomly with probabilities Prob(1) = 0.7 and Prob(0) = 0.3. What is the probability that at least three 0's will be generated in a sequence of five digits. Prob(i) represents probability of generating the digit 'i'.

   (b) The probability density function of amplitude of a random signal follows uniform distribution in the range $(-A, A)$. The signal is applied to a half-wave rectifier circuit. Assuming an ideal diode, find cumulative distribution function and probability density function (PDF) of output signal amplitude. Plot the PDF and CDF curves.

   [4+6]

7. Prove that if input to a liner time-invariant system is a wide-sense stationary (WSS) process, then output of the system is also a WSS process.

   [10]

8. (a) Prove that autocorrelation function $R_x(\tau)$ of a wide sense stationary random process $X(t)$ satisfies the following.

   $$|R_x(\tau)| \leq R_x(0)$$

   (b) PSD of a stationary random process $X(t)$ is given by $Sx(f)$. Random process $Y(t)$ is defined by $Y(t) = X(t) \cos(2\pi f_c t + \Theta)$

   Where Θ is a random variable uniformly distributed over $(0, 2\pi)$. Determine the expression of PSD $S_Y(f)$ of $Y(t)$ in terms of $Sx(f)$. Assume $X(t)$ and Θ are independent of each other.

   [5+5]