B.E. (ETC) Part-II 4th Semester Examination, 2010

Mathematical Techniques

(MA-401)

Time : 3 hours **Full Marks : 70**

Use separate answerscriot for each half.
Answer SIX questions, taking THREE from each half.
Two marks are reserved for general proficiencu in each half.

FIRST HALF

1. a) Find the inverse of a matrix A by partitioning ,

$$\text{where } A = \begin{bmatrix} 3 & 2 & 1 \\ I & 1 & I \\ 5 & 1 & -1 \end{bmatrix}$$

b) Show that the eigenvalues of a Hermitian matrix are all real. |6+5J

2. a) Find the eigenvalues and corresponding eigenmeters of the matrix

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

b) Finding the algebraic multiplicity and geometric multiplicity of the eigenvalues of the above matrix. Verify the statement "geometric multiplicity can not be greater than algebraic multiplicity". [5+6|

3. a) Prove that **<a, cP** + dy> = c **<a, P>** + d **<a,** y).

b) Find the value of K so that u = (I, 2, K, 3) and v = (3, K, 7, -5) in R^4 are orthogonal.

c) Define the norm of a vector u in an Important space and show that for any scaler C, ||Cu|| = |C| ||u[|.

d) State Cauchy-Schwarts inequality in a complex inner product space.

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4. a) Trie eigenvalues of a square matrix A of order (n + 1) are 0 and the nth roots of.unity. Prove that

$$\frac{}{21 - A} = I + \frac{}{2 - I} - s\, i — [2^{n-1} A + 2^{n-2} A^2 + \ldots + 2A^{n-1} + A^n]$$

b) Let A = [! **J],**

Verify that A is unitarily diagonalizable. |5+6]

5. a) Consider complex vector space C^3 with Euclidean innerproduct. Apply Gram-Schmidt process to convert the basis **Uj** = (i, i, i), u_2 = (0, i, i), u_3 = (0, 0, i) into an orthogonal basis.

b) Find the rank, index and signature of the quadratic form.
$2x^2 + 5y^2 + 10z^2 + 4xy + 6zx + 12yz$.
Also discuss the definiteness of the quadratic form. |6+5]

SECOND HALF

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6. a) Use known Laplace transforms and/or transform properties to evaluate

r COS(t,) ,
J - ? T T " d x - t > 0 -
o

b) Show that

F c {cos (0 ^ } = ^ _f cos - ^t) dt

and hence deduce that

F_c [cos ; (\ = a cos (^) + b sin (^),

where a and b are constants.

c) If Fft) = F_c [$e^{-t^2/2}$; prove that

F(0)=1

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and hence show that F(x) = e s . (4+4+31

7. Solve the following boundary value problem using Laplace transform technique :

ae(x,t) = c 2 **£ e (x . t)** 0 < x < o o j t > 0 >
dt *ox*

$6_x(0, t) = -f(t)$, t > 0, 9(x,0) = 0, 0<x<ooand G(x,t)->0 when x-» «>.

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8. a) Apply Laplace transform technique to solve the following initial value problem:
y"(t) + 4y(t) = sint,
y(0) = y'(0) = o
where ' denotes derivative with respect to t.

b) Use Laplace transform to solve the following integral equation :

$$f(t) = 1 + 2 \int_{0}^{t} f(x) \sin(t - T)\, dx, \; t > 0.$$ [5+6]

9, a) State and prove the Convolution Theorem for Laplace transforms.

b) Light travels in a medium from one point to another so that the time to travel given by J" $_v$(x^{S}y) 'S m m ' m u m - (Hence s is the arc length and v(x, y) is the velocity of light in the medium). Show that the path of travel is given by

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10. a) Determine the shape of the wire so that a particle sliding from a fixed point on the wire reaches the other point in least possible time.

b) Find the extrema of the function

$$v(x) = \int_{o} (x_1^2 + x_2 + 2x_1 x_2)\, dt$$

subject to the boundary conditions

x,(0) = 0, x, (!) = !, $x_2(0) = 0$ and x_2(§) = -!. I6+5J