B.E (ETC) 7th Semester Final Examination, 2011

Sub: Digital Signal Processing (ET 704)

Full marks: 70 **Time: 3 hours**

Answer any Five questions

1. a) Determine whether each of the following signals is periodic. If the signal is periodic, state the period.

   i) $x[n] = \exp(j\pi n/6)$, (ii) $x[n] = [\sin(\pi n/5)]/(\pi n)$

   b) The accumulator system is defined by the input-output equation

   $$y[n] = \sum_{k=-\infty}^{n} x[k]$$

   i) Is it a linear system? ii) Is it a BIBO stable system?

   c) Determine the DTFT of $x[n]$, where $x[n] = a^n u[n]$, $|a|<1$ 14

2. a) Impulse response of an LTI system is given by $h[n] = (0.5)^n u[n+2]$. Is it a causal system?

   b) Analyze the block diagram of Fig.-1 and develop the relation between $y[n]$ and $x[n]$.

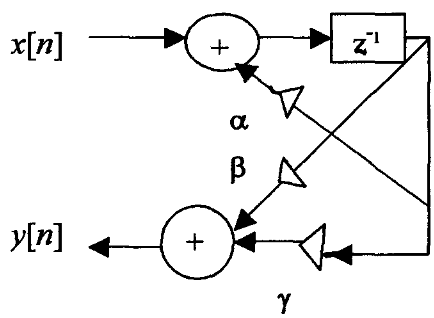

Fig.-1

   (c) Let $x[n]$ and $y[n]$ denote complex sequences and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective DTFTs. Show that

   $$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$$

   where $y^*[n]$ and $Y^*(e^{j\omega})$ represent complex conjugates of $y[n]$ and $Y(e^{j\omega})$ respectively. Hence, prove

   $$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$ 14

3. a) If N-point DFT samples of an N length sequence $\{x[n]\}$ $(0 \le n \le N-1)$ is given by sequence $\{X[k]\}$, $0 \le k \le N-1$, then prove that

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]exp(j2\pi kn/N), \qquad 0 \le n \le N-1$$

b) Prove that an LTI causal system will be stable if and only if its poles lie inside the unit circle.

c) Determine Z transform of the following sequence.

$$x[n] = (2)^n\, u[n] + (0.5)^n\, u[-n-1]$$ 14

4. a) Determine inverse Z transform of $X(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-2}}$, ROC: $|z| > 1/2$

b) Draw the magnitude response of the digital filter characterized by the following transfer function. $H(z) = 0.5\,(1 + z^{-1})$

Find out an expression for the 3-dB cutoff frequency of the digital filter obtained by cascading M-stages of the above filter. 14

5. a) Prove that an FIR filter of order 6 will have a linear phase response if its impulse response $h[n]$ is symmetric, i.e.

$$h[n] = h[6-n], \qquad 0 \le n \le 6$$

b) Determine the impulse response coefficients $h[n]$ of a causal FIR filter of length 3, if the filter passes a cosine discrete-time sequence of angular frequency of 0.1 rad/samples and completely blocks a cosine discrete-time sequence of angular frequency of 0.5 rad/samples. Assume $h[0] = h[2]$. 14

6. a) Determine and plot impulse response coefficients $h[n]$ of a causal FIR filter of length 3 with cutoff frequency 800Hz and sampling rate of 8,000 Hz using window technique. Use Hanning window.

b) Discuss the periodogram technique of estimating power spectral density of a wide sense stationary discrete time random process. 14

7. a) What is Gibbs phenomenon. How can it be controlled?

b) Passband and stopband edge frequencies of an IIR lowpass digital filter are 250Hz and 550 Hz respectively. The sampling frequency is 2000Hz. Maximum passband ripple is 0.6 dB and minimum stopband attenuation is 16 dB. Determine the order of the filter using butterworth approximation. Deduce necessary relations. 14

8. a) Explain how MATLAB® plots (i)frequency spectrum of a given finite length discrete-time signal and (ii) frequency response of an IIR LTI system.

b) Draw a parallel form structure to realize the following transfer function.

$$H(z) = \frac{1}{(1 - 2.5z^{-1} + z^{-2})}$$ 14