BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR
B.E. 3RD SEMESTER (ETC) FINAL EXAMINATIONS, 2010
Signals and Systems (ET 301)

Time: 3 hours **Full marks: 70**

Use Separate Answer Scripts For Each Half

First Half

Answer Question No. 1 And Any Two From The Rest

1. a) Prove that if input to a liner time-invariant system is a wide-sense stationary (WSS) random process, then output of the system is also a wide-sense stationary random process.

b) A running integrator is defined by

$$y(t) = \int_{t-T}^{t} x(\tau)d\tau$$

where $x(t)$ is the input, $y(t)$ is the output, and T is the integration period. Both $x(t)$ and $y(t)$ are sample functions of stationary processes $X(t)$ and $Y(t)$, respectively. Show that the power spectral density (PSD) $S_Y(f)$ of the integrator output is related to that of the integrator input $S_X(f)$ as

$$S_Y(f) = T^2 \operatorname{sinc}^2(fT)\, S_X(f)$$

c) If $m_h(t)$ is the Hilbert transform of $m(t)$, then show that Hilbert transform of $m_h(t)$ is $-m(t)$.

7+6+2

2. a) Prove that autocorrelation function $R_x(\tau)$ of a wide sense stationary random process $X(t)$ satisfies the following.

$$|R_x(\tau)| \le R_x(0)$$

b) Show that two random variables **x** and **y** are related by $\mathbf{y} = k_1\mathbf{x}+k_2$, where k_1 and k_2, are two arbitrary constants, the correlation coefficient $\rho_{xy} = 1$, if k_1 is positive, and $\rho_{xy} = -1$ if k_1 is negative.

5+5

3. a) If **x** and **y** are two uncorrelated random variables and $\mathbf{z} = \mathbf{x} + \mathbf{y}$, prove that

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 .$$

where $\sigma_X^2, \sigma_Y^2, \sigma_Z^2$ are variances of random variables **x**, **y** and **z** respectively. Derive necessary results.

b) Explain the importance of Wiener-Khinchine theorem in frequency domain characterization of a stationary random process.

5+5

4. a) The joint probability density function (PDF) of two continuous random variables **x** and **y** is represented by $p_{xy}(x,y)$. Prove that PDF of **y** is given by

$$p_y(y) = \int_{-\infty}^{\infty} p_{xy}(x, y)dx$$

b) In a random experiment, a trial consists of five successive tosses of a coin. If we define a random variable **x** as the number of tails appearing in a trial, determine the cumulative distribution function of **x** and plot it.

5+5

2nd Half

Answer question No. 5 and any other two from 2nd half.

Q 5. 15

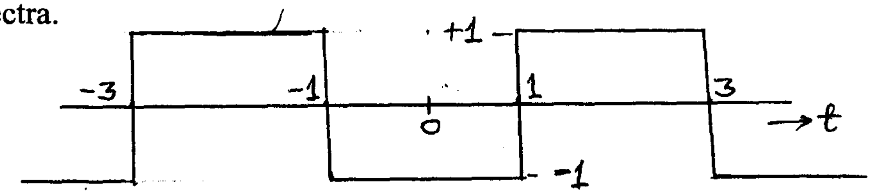
i) State and prove Parseval's theorem
ii) State and establish Nyquist sampling theorem
iii State and prove Fourier's theorem

Q 6. What do you understand by 10

i) Energy signal and power signal
ii) Energy spectral density and Power spectral density

Prove that autocorrelation function and energy spectral density are Fourier transform pair. Prove that Fourier transform of convolution of time dependent functions is given by the product of the Fourier transform of individual function.

Q 7. Find the Fourier series of periodic signal shown below and sketch the amplitude and phase spectra. 10



Explain the condition for a system to qualify as a Linear Time Invariant (LTI) system
A system has the input –output relationship given by

$$y = T[x] = x^2$$

Show that system is non linear
Determine whether the signal $x(t) = e^{-\alpha t}u(t)$ $\alpha \rangle 0$ is an energy signal or power signal.

Q 8. Sketch the following discrete signal 10

$i)$ $x[2n]$ $ii)$ $x[-n+2]$

b) Determine whether or not each of the following signals is periodic. If this is periodic determine the fundamental period

$$i) \quad x[n] = \cos\frac{\pi}{8}.n$$

$$ii) \quad x[n] = \cos^2\frac{\pi}{8}.n$$

c)A system has the input-output relation given by

$$y[n] = T\{x[n]\} = nx[n]$$

Determine whether the system is memoryless, causal, linear and time invariant.