

Use separate Answer Script for each Half.

First Half

Answer Any Three questions.

Two marks are reserved for general proficiency in this half.

Symbols have their usual meanings.

1. i) State division algorithm.

ii) Using division algorithm prove that square of an odd integer is of the form $8k + 1$.

iii) If $\gcd(a,b) = d$ then prove that a/d and b/d are integers prime to each other.

2+4+5=11

2. i) Prove that the numbers of primes is infinite.

ii) If $ac \equiv bc \pmod{m}$ and $\gcd(c,m) = d$ then prove that $a \equiv b \pmod{m/d}$.

iii) Show that

$$7 \text{ divides } 2222^{5555} + 5555^{2222}$$

4+3+4=11

3. i) If p be a prime and k be a positive integer then prove that

$$\phi(p^k) = p^k \left(1 - \frac{1}{p}\right) \text{ where } \phi \text{ is Euler phi function.}$$

ii) Find the number of integers less than n and prime to n when $n = 900$.

iii) Show that $n^2 + 3n + 5$ is never divisible by 121 for any positive integer n .

4+2+5=11

4. i) Find the remainder when 41^{75} is divided by 3.

ii) Show that a tree with n vertices has $n-1$ edges.

iii) A certain tree T of order 21 has only vertices of degree 1, 3, 5 and 6. If T has exactly 15 end vertices and one vertex of degree 6, how many vertices have degree 5?

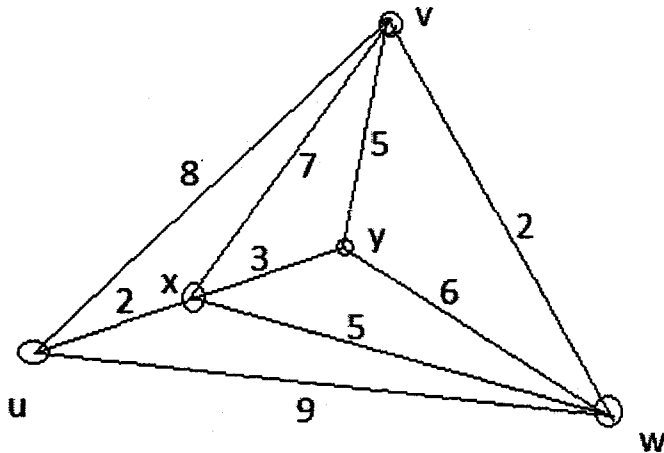
$$3+4+4=11$$

5. i) Define spanning tree of a connected graph. Give an example.

ii) Discuss Kruskal's algorithm to find a minimum spanning tree of a connected weighted graph.

iii) Apply Kruskal's algorithm to find a minimum spanning tree of the following weighted graph

$$2+5+4=11$$



SECOND HALF

Answer Question No. 6 and any TWO from the rest

6. Compute $f'(1.6)$ and $f''(1.6)$ for the function $y=f(x)$, given in the table:

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
f(x):	7.989	8.403	8.781	9.129	9.451	9.750	10.031

after deducing the associated formula. [13]

7. a) Obtain an approximate value of $\int_0^1 \frac{dx}{1+x^2}$ up to four places of decimal by using Simpson's $\frac{1}{3}$ rule, taking four equal sub-intervals of $[0, 1]$ and hence obtain the approximate value of π correct to four decimal places.

- b) Calculate $f(5)$ from the following data:

x:	2	4	7	9
f(x):	10	26	65	101

[6+5]

8. After deducing Gauss's forward formula, find the value of $e^{1.17}$ from the following table:

x :	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x :	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

[11]

9. a) Derive Predictor-Corrector formula which uses forward differences in Milne's method to solve the first order differential equation $\frac{dy}{dx} = f(x, y)$ with the initial condition $y(x_0) = y_0$.

- b) Using Runge-Kutta method of order 4, find $y(0.2)$ for the initial value problem

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$$

taking $h=0.2$.

[6+5]