Bengal Engineering and Science University, Shibpur
B.E. (CST) 7th Semester Examination 2012
Graph Algorithms (CS – 705/5)

Time: 3 hours   **Full marks: 70**

Attempt questions from both halves

FIRST HALF
*Attempt question 1 and any two from the rest*

1. (a) Define a *flow* over a given directed graph G with source node s, sink node t, and a capacity function c which assigns a positive capacity to every edge in G.
(b) Prove or disprove the following statements, giving suitable justifications:
(i) A connected planar graph must have at least one node of degree less than 6.
(ii) If a connected graph does not contain any cycle, then it is planar.   [4 + (4 + 3)]

2. (a) State the Gale Shapley algorithm for stable marriages.
(b) Use the Gale Shapley algorithm to solve the following instance of the stable marriage problem. Show all steps in the solution.
There are four men (M1, M2, M3, M4) and four women (W1, W2, W3, W4); their preferences are as follows:

|			
M1:	W4, W1, W2, W3	W1:	M4, M1, M3, M2
M2:	W2, W3, W1, W4	W2:	M1, M3, M2, M4
M3:	W2, W4, W3, W1	W3:	M1, M2, M3, M4
M4:	W3, W1, W4, W2	W4:	M4, M1, M3, M2

[4 + 8]

3. (a) State the algorithm by Demoucron to detect the planarity of a given graph.
(b) Use the above algorithm to detect whether the following graph is planar, showing all steps.
(Hint: You can start the algorithm considering the cycle ABCIFEA).   [4 + 8]
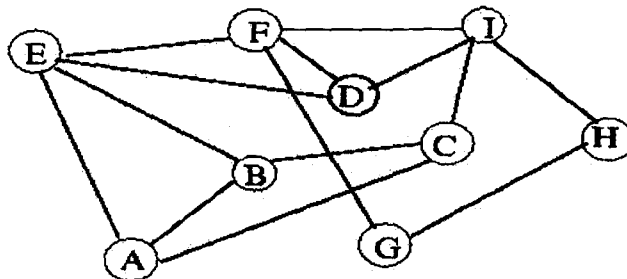

4. (a) Briefly describe the following topological properties of networks: (i) degree distribution, (ii) clustering coefficient.
(b) Describe algorithms to find the bow-tie structure of a given connected directed graph, (i) when the graph is small, having few tens of nodes, and (ii) when the graph is large, of the scale of the Web graph.   [4 + (4 + 4)]

5 a) Define connected component of a graph. If G(V,E) is a simple graph with n vertices and k connected components, then find the maximum number of edges in G.   5.5

b) Define strongly connected components (SCCs) of a graph. State Kosaraju's algorithm to find SCCs of a graph. Find the SCCs of the following graph with Kosaraju's algorithm.   12
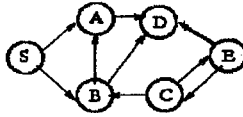

6 a) Define *interval* graph. Extracts intervals from the following graph. Check whether the graph is an interval graph or not. Let the edge between `B' and `b' of the graph is removed to form G'. Is G' an interval graph?   10
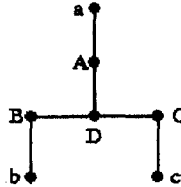

b) Define induced subgraph H of a graph G. Find (i) 3-edge induced subgraphs, (ii) 3-vertex induced subgraphs, (iii) a subgraph H that is edge induced but not the vertex induced, and (iv) a subgraph HH that is vertex induced but not the edge induced, of the following graph.   7.5
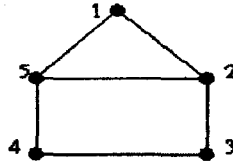
7 a) State Luby's algorithm to find MIS (maximal independent set). Find out MIS for the following graph showing steps of Luby's algorithm, considering 2-processor (i) EREW and (ii) CRCW SIMD m/c.   12
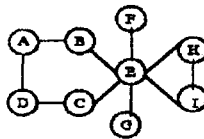
b) Define clique matrix. Find clique matrix of the following graph. Check whether it follows the consecutive 1s property.   5.5
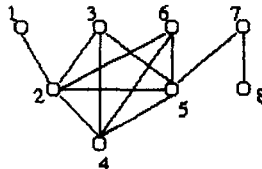
8. Write short notes on the following: 12+5.5
a) Floyd-Warshall algorithm to find all-pairs shortest paths.
b) Chordal graph