Bengal Engineering and Science University, Shibpur

BE (CST) 6th Semester End-Term Examination, May 2012

Department of Computer Science and Technology

Theory of Computation (CS - 602)

Full Marks: 70 Time : 3 Hours

- **Attempt any five questions.**
- **All questions carry equal marks.**
- **Answers should be in your own words as far as practicable.**
- **Make your own assumptions if necessary and state them at proper places.**

1. (a) What do you mean by a language L over an alphabet Σ?

   (b) What do you mean by finite representation of a language? Prove that not all languages over some finite non-empty alphabet Σ have finite representations.

   (c) What are the finite representations of languages you are aware of? State strength and limitation of each of those representations.

   (d) Give an example of a string which is in and a string which is not in the following languages, where $\Sigma = \{a, b\}$.

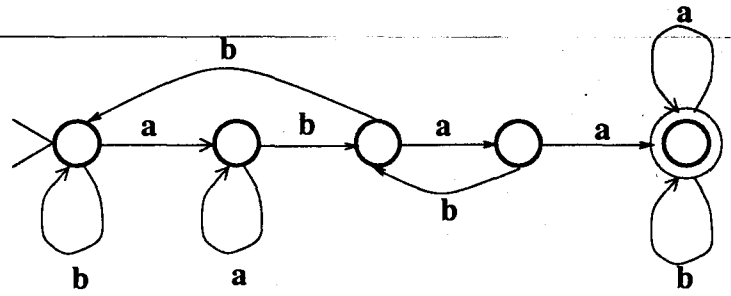
   i. $\{\omega : \omega\omega = \omega\omega\omega\}$

   ii. $\{\omega :$ for some u and v, $uv\omega = \omega vu\}$ [2+4+4+4]

2. (a) In the context of non-deterministic finite automata $E(q)$ denotes the empty closure of a state q. Write an algorithm that, given a non-deterministic finite automata M and a state q in it, determines $E(q)$.

   (b) Given a deterministic finite automata M, state how you can obtain a regular expression R so that $L(R) = L(M)$. Obtain the regular expression corresponding to the finite automata shown below.

   [4+10]



3. Let M be a **special** finite automaton defined as a quintuple $(K, \Sigma, \delta, s, F)$, where K, Σ, s are as described for deterministic or non-deterministic finite automata and δ is a function from a finite subset of $K \times \Sigma^*$ to K. Formally define the following.

   (a) Configuration of M.

   (b) The relation $\vdash_M$

   (c) The language accepted by M.

   Is a **special** finite automaton more like a deterministic or a non-deterministic finite automaton? Formally justify your answer. Give an example of a language which is accepted by a **special** finite automaton.

   [6+4+4]

4. Show that the following languages are context-free by exhibiting context free grammars generating each.

   (a) The language of all odd-length palindromes over $\{a, b\}$. A string is a palindrome if it reads same from both the ends.

   (b) $\{a^n x : n \geq 0, x \in \{a, b\}^* \text{ and } |x| \leq n\}$

   (c) $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } i = j \text{ or } j = k\}$

   (d) $\{a^n b^m : n \leq m \leq 2n\}$ [3+4+4+3]

5. Show that if a language is context-free, then it is accepted by a push-down automaton. Adopt a top-down approach to present the proof, that is, first give a sketch for the main steps in the proof with subjective justification and then refine those steps. [14]

6. Use pumping theorem to show that the following languages are not context free.

   (a) $\{a^n b^{2n} a^n : n \geq 0\}$

   (b) $\{a^{n^2} : n \geq 0\}$ [2×7]

7. Formally define a **Machine Schema**. Is machine schema more powerful than standard Turing Machine? Formally justify your answer. Explain with examples how it is more advantageous to use machine schema rather than standard Turing machine. [3+8+3]

8. Write short notes on the following.

   (a) Turing decidable languages

   (b) Extensions of Turing machine

   (c) Properties of an automaton

   (d) Grammar to compute functions [3+4+3+4]