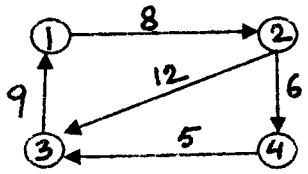
Bengal Engineering and Science University, Shibpur
B.E. 5th Semester Examination, 2012
Design and Analysis of Algorithms (CS-504)

Time:3 hours **Full Marks:70**

Attempt Question no. 1 and any FIVE from the rest.

1.a) Give the adjacent list and adjacent matrix representation of the following weighted graph.



(10)

b) Mention two major disadvantages of the "Division Method" used for making a hash function.
c) Define the terms "Spanning Tree" and "Spanning Forest".
d) How many exchanges are required to sort the numbers {3, 9, 7, 11, 19, 4, 15} in ascending order, using selection sort algorithm?
e) Define "In place sort" and "Stable sort" and give one example of both type of sorting.

2.a) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove the following: (6)

(i) $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.

(ii) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

b) Let $a \geq 1$ and $b > 1$ be constants and $f(n)$ be a nonnegative function defined on exact powers of b. A function $g(n)$ defined over exact powers of b by

$$g(n) = \sum_{j=0}^{\log_b n - 1} a^j f(n/b^j)$$

If $f(n) = O(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$ then prove that $g(n) = O(n^{\log_b a})$. (6)

3.a) Construct the recurrence tree for the recurrence $T(n) = a\,T(n/b) + f(n)$; $a \geq 1$ and $b > 1$, where $f(n)$ be a nonnegative function and $T(n)$ be defined on the nonnegative integers. Also do the following:

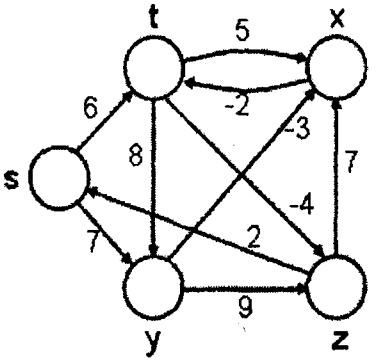
- Compute the height of the tree
- Count the number of leaf nodes of the tree
- Find out the solution of the above recurrence (8)

b) Give the recurrence equation corresponding to the Binary search algorithm and solve it using Master's method. (4)

4.a) Write down the Randomized version of quicksort algorithm and demonstrate it with an example. (6)

b) Prove that the average case running time of this quicksort algorithm is $O(n\log_2 n)$. (6)

5.a) Let G = (V, E) be a connected, weighted graph. Define Bellman-Ford algorithm BELL_FORD (V, E, W, s) for finding the shortest path from source vertex s to all other vertices.
Illustrate the algorithm using following graph and compute the shortest path from s to other vertices. (8)



b) Let G = (V, E) be a weighted, directed graph with source s. Prove that, if G contains negative weight cycle reachable from s, then the algorithm return 'FALSE' (means no solution). (4)

6.a) What is clique? Define a clique problem both as optimization and decision problem. (4)

b) Prove that the clique problem is NP-Complete. (8)

7.a) Define 0 - 1 Knapsack problem. Give sufficient justification based on which one can conclude that dynamic programming can be used to solve it. (6)

b) Write down Huffman code generation algorithm.
Construct Huffman code for the characters present in a 100-character data file, characters and their frequencies in the file is listed in following table. (6)

Characters	A	B	C	D	E	F
Frequencies	45	13	12	16	9	5

8.a) Discuss at least three issues regarding the design of good hash functions. (4)

b) In a hash table in which collisions are resolved by chaining, prove that, under the assumption of simple uniform hashing, the average time required for (8)

(i) an unsuccessful search is $\Theta(1+\alpha)$ and

(ii) a successful search is $1+\frac{\alpha}{2}-\frac{1}{2m}$.

[α = n/m is the load factor]