B.E (C.S.T) 5^{th} Semester Exam, 2011

Mathematics-v (MA-501)

Time: 3hrs **Full Marks: 70**

Use Separate Answer script for each half.

Answer six questions taking three from each half

Two marks are reserved for general proficiency in each half.

Symbols have their usual meaning.

First Half

1. i) State and prove Euler's Handshaking Lemma.

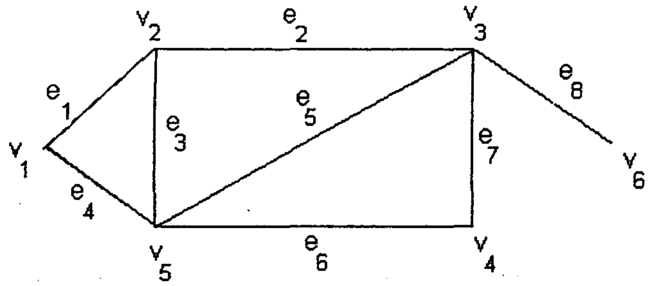
   ii) Can there be a simple infinite graph with finite number of vertices?

   iii) Define a K-regular graph? Show that if G is a K-regular graph with n vertices and e edges then $\delta = \mathbf{2e/n} = \Delta$.

   5+2+4=11

2. i) Define spanning subgraph and induced subgraph. Give examples.

   ii) Give examples of walk of length **3** and length **4** in the following graph.



   iii) The degree of every vertex of a graph G of order 25 and size 62 is 3, 4, 5 or 6. There are 2 vertices of degree 4 and 11 vertices of degree 6. How many vertices have degree 5 ?

   3+4+4=11

3. i) Using principle of induction, show that 3^{2n} -8n -1 is divisible by 64.

   ii) Using Division algorithm, show that product of any n consecutive integers is divisible by n.

   iii) State Euclidean algorithm. Use it to obtain integers u and v such that

   $$\gcd(13,80) = 13u + 80v$$

   4+3+4=11

4. i) If $ac \equiv bc \pmod{m}$ and $\gcd(c, m) = 1$ then show that

   $$a \equiv b \pmod{m}$$

   ii) Show that $3.4^{n+1} \equiv 3 \pmod{9}$ for all positive integer n.

   iii) Show that N = 35078571 is divisible by 11.

   4+4+3=11

5. i) State and prove Fermat's Little Theorem.

   ii) Using alternative definition of gcd and LCM, find gcd(40,24) and LCM(54,50).

   iii) Find the number of integers less than n and prime to n where n = 324.

   5+3+3 =11

SECOND HALF

6. a) Let f(x) be continuous and have continuous derivative of order (n+1) for all x in an interval I containing the interpolating points $x_0, x_1, x_2, x_3, \ldots, x_n$. Then at any point x on I, prove that the error in approximating f(x) by the interpolating polynomial g(x) is given by

$$R(x) = f(x) - g(x) = \frac{w(x) f^{n+1}(c)}{(n+1)!}$$

where c is a point in the interval I and w(x) is given by

$$w(x) = (x-x_0)(x-x_1)(x-x_2)\ldots(x-x_n).$$

b) Calculate f′(0.4) using the table

x:	0.3	0.5	0.6
f(x):	0.61	0.69	0.72

[6+5]

7. Compute f′(0.2) and f″(0.2) for the function y=f(x), given in the table:

x:	0	1	2	3	4	5	6
f(x):	1	8	35	94	197	356	583

after deducing the associated formula. [11]

8. a) Construct the interpolating cubic spline with free boundary conditions.

b) Discuss Gauss-Seidel iteration method by considering a system of n linear equations with n variables $x_1, x_2, x_3, \ldots, x_n$ viz.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n. \end{aligned}$$

[7+4]

9. The following table gives the specific heat of ethyl alcohol at different temperatures. Estimate the specific heat corresponding to 15^0C.

Temp. (x^0C):	0	10	20	30	40	50
Sp. Heat(y) :	0.51	0.55	0.57	0.59	0·62	0.67

[11]

10. a) Derive Predictor-Corrector formula which use forward differences in Milne's method to solve the first order differential equation $\frac{dy}{dx} = f(x, y)$ with the initial condition $y(x_0) = y_0$.

b) Use Milne's method to solve the following differential equation

$$\frac{dy}{dx} = 1 + y^2 \text{ with } y(0) = 0$$

and compute y(0.8), given that y(0.2) = 0.2027, y(0.4) = 0.4228, y(0.6) = 0.6841.

[6+5]