B.E. (CST) Part-II 4th Semester Examination, 2010

Probability and Statistics

(MA-402)

Time ; 3 hours | Full Marks : 70

Use separate answerscript for each half.

Answer SIX questions, taking THRBB from each half.

Two marks are reserved for general proficiencu in each half.

FIRST HALF

1. a) Prove that for 3 events A, B and C,

P(AuBuC) = P(A)+P(B) + P(C)-P(AnB)-P(BnC)

-P(Cr»A) + P(Ar>Br»C).

b) Use induction to generalize the above result to n events (n is a positive integer).

c) Calculate the probability of 'exactly k matches* in the context of a matching problem. I3+3+5J

2. a) There are 37 red balls and 52 black balls in a box. 46 balls are drawn at random from the box. What is the probability that 29 of them are red? If X be the number of red balls drawn, calculate E(X).

b) A real-valued random variable X has probability density function (p.d.f.) defined by , . .

$$f(x) = \frac{1}{\sqrt{2\pi}\ 3}\ e^{-\frac{\quad}{18}}$$, — QO < X < 00.

Find $E(e^{tx})$ and $Var(3X + 73)$. |5+6|

3. a) If X and Y are stochastically independent random variables, show that

E(XY) = E(X)E(Y)

Would the result $E(2X^3 \sin(2Y))$ - $4E(X^3)E(\sin Y \times \cos Y)$ be also true? Justify your answer.

b) X, Y and Z are real-valued random variables such that E(X) = E(Y) = E(Z) = 1. If Var(X) = Var(Y) = Var(Z) = 4, find $E(X^2 + 2Y^2 + 9Z^2)$. Are the random variables necessarily mutually independent? Is Cov(X, Y) = 12 possible? Explain. [6+5]

4. a) U_1 and U_2 are independent u(0, 1) random variables. Suppose

$$X = \sqrt{-2\log_e U_1}\ \cos(2\pi U_2)$$

and $$Y = \sqrt{-2\log_e U_1}\ \sin(2\pi U_2)$$

Prove that X and Y are independent N(0. 1) random variables.

b) Discuss the 'mixed congruential method* for generating pseudorandom numbers. 18+3]

5. a) Suppose h(x, y) = f(x) g(y)

where f(x) and g(y) are both p.d.f.s. Show that

(i) h(x, y) is a bivariate p.d.f.

(ii) h(x, y) has marginal densities given by f(x) and g(y) respectively.

(iii) The random variables X and Y whose densities are f(x) and g(y) are stochastically independent.

b) Suppose X~u(0,1). Does $E(\log_e X)$ exist? If so, find it. [7+4]

SECOND HALF

6. a) $(X_1, X_2, \ldots, X_n)$ is a random sample from U(0, 9), 9>0. Find the M.L.E. of 9.

b) $(x_1, x_2, \ldots, x_n)$ is a random sample from $N(u, a^2)$. Explain how would construct a 95% confidence interval for "u" when

(i) a is known,

(ii) a is unknown. (4+71

7. a) Define unbiased estimator. Prove that for a ransom sample $(x_1, x_2, \ldots, x_n)$ of size n, taken from an infinite population, the statistic S^2 defined by $S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is not unbiased estimator of population variance a^2, but the statistic s^2 defined by $s^2 = \frac{n}{n-1} S^2$ is an unbiased estimator of a^2.

b) Obtain 95% confidence interval for the population parameter X of the Poisson distribution :

f(x,X)= *~^> x = 0> U2, ... [6+51

8. a) What do you meant by central tendency of data? What are different measures of it? Discuss relative advantages and disadvantages of it?

b) What dp you understand by correlation between two random variables? Find the correlation coefficient between X and Y based on following data :

X	1	2	3	4	5	6
Y	5.1	6.8	8.9	11.2	13	15.1

9. a) Define critical region, Type-1 error, Type-II error and power of test in case of testing of the Hypothesis $H_0: u = u_0$ against $H_a : u * u_0$.

b) If W = {x : x2:1} is the critical region for testing $H_0 : 0 = 2$ against the alternative $H_a : 0 = l$, on the basis of the single observation from the population, f(x, 8) = 8exp(-8x), 0<x<oo.
Obtain the values of Type-I and Type- II errors. (6+51

i

10. a) State and prove Neyman-Pearson Lemma in connection with testing of a hypothesis.

b) Use Neyman-Pearson Lemma to obtain the best critical region for testing the hypothesis $H_0 : 8 = 8_0$ against $H_a : 8 = 8] > 0o$ in case of a normal population $N(8, a^2)$ where a^2 is known. |6+51