B.E. (CST) Part-II 4th Semester Examination, 2007

Probability and Statistics

(MA-402)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.

Answer SIX questions, taking THREE from each half.

Two marks are reserved for general proficiency in each half.

FIRST HALF

1. a) In a certain examination, six papers are set, and to each are assigned 100 marks as a maximum. A student is assumed to get any score out of 100 with equal chance. Show that the probability of his/her getting forty percent of the whole number of marks is

$$\frac{\frac{1}{5!}\left\{\frac{(245)!}{(240)!} - 6\,\frac{(144)!}{(139)!} + 15\,\frac{(43)!}{(38)!}\right\}}{(101)^6}$$

b) State and prove Bayes' theorem. [7+4]

2. a) What do you understand by the term "matching problem"? Discuss.

b) If X denotes the number of matches in a "matching problem", find E(X) and Var(X). [4+7]

3. a) Find the moment generating function (m.g.f.) of a binomial random variable with mean np (and variance np(1 − p)). Show that if X ~ Bin (n, p) and Y ~ Bin (m, p) and X and Y are stochastically independent, then X + Y ~ Bin (n+m, p).

b) Consider the distribution having p.d.f. given by

$$f(x) = \frac{1}{\pi}\cdot\frac{1}{1+x^2},\ -\infty < x < \infty.$$

Show that this distribution does not have finite mean. Does the m.g.f. exist for this distribution? Give reasons for your answer. [6+5]

4. a) A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that takes him to safety after 2 hours of travel. The second door leads to a tunnel that returns him to the mine after 3 hours of travel. The third door leads to a tunnel that returns him to the mine after 5 hours of travel. Assuming that the miner is at all times equally likely to choose any one of the doors, find the m.g.f. of X, the time when the miner reaches safety. (Note that our assumption implies that the miner is stupid!)

b) Deduce the normal equations for a linear regression model, starting from first principles. [6+5]

5. a) Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables such $X_i \sim N(\mu, \sigma^2)\ \forall\ i = 1, 2, \ldots, n$. Show that

i) $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

ii) $\sum_{i=1}^{n} (X_i - \bar{X})^2/\sigma^2$ is chi-square distributed with $(n-1)$ degrees of freedom.

b) Prove the formula :

$$\text{Var}(X_1 + \ldots + X_n) = \sum_{i=1}^{n} \text{Var}(X_i) + 2 \sum_{i<j} \sum \text{Cov}(X_i, X_j).$$

What does this formula reduce to, when the X_i's are independent random variables? Justify your answer. [6+5]

SECOND HALF

6. a) Calculate mean and median of the following frequency distribution :

Class-interval	0 – 8	8 – 16	16 – 24	24 – 32	32 – 40	40 – 48
Frequency	8	7	16	24	15	7

b) Write short notes on :

(i) Frequency Polygon, (ii) Skewness. [6+5]

7. a) Define consistent and unbiased estimator.

Let $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \Theta$,

i) $E_\theta(T_n) \to \theta$; $n \to \infty$

and ii) $\text{Var}_\theta(T_n) \to 0$ as $n \to \infty$.

Then show that T_n is a consistent estimator of θ.

[Θ denotes the parametric space].