B.E. (CST) Part-II 3rd Semester Examination-2011

**Mathematics-IIIC (MA-303)**

Time-3 hours | Full Marks: 70

Use separate answerscript for each half.

Answer SIX questions, taking THREE from each half.

Two marks are reserved for general proficiency in each half.

## FIRST HALF

1(a) Define analytic function.

(b) Prove that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied at that point.

(c) If f (z) = u + iv is an analytic function of z then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

[2+4 +5]

2. State and prove the Cauchy integral formula for the derivative of an analytic function f (z). Evaluate the integral $\int_0^{1+i} z^3 dz$ along the paths (i) $y = x^2$ (ii) $y^2 = x$

[(1+4)+(3+3)]

3(a) If $F(a)=\int_c \frac{(z^2+z+1)}{z-a}dz$, where C is a positively oriented circle $x^2+y^2=9$. Find F (4) and F(2i).

(b) Find the Taylor's or Laurent series in powers of z which represent the function $f(z) = \frac{z^2+z+1}{(z-2)(z-3)}$ in the regions

i) $|z| < 2$ ii) $2 < |z| < 3$ iii) $|z| > 3$

[5+2×3 ]

4. Define residue of a function f(z) at the isolated singular point $z_0$ . Using the method of contour integration evaluate the following integrals (Any two):

i) $\int_{-\infty}^{\infty} \frac{\cos mx dx}{(x^2+a^2)(x^2+b^2)}$ ii) $\int_{0}^{\infty} \frac{\sin mx}{x} dx$ iii) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)^2} dx$

[1+5+5]

**5**(a) Define Fourier Series of a function f(x) of period T . Find the half range expansion in a cosine series of the function $f(x) = kx,\ 0 \le x \le \frac{l}{2}$

$= k(l-x),\ \frac{l}{2} \le x \le l$.

Hence deduce the sum $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

(b) Solve and give a physical interpretation to the following Boundary Value Problem by separation of variables technique.

$u_t = c^2 u_{xx}$

where

$u(0,t) = 0,\ u(l,t) = 0$

$u(x,0) = f(x)$.

[(1+4)+6]

~0~

# SECOND HALF

Q.6.a) Let G be a set of all real numbers except$-1$. Define $*$ on G by

$a * b = a + b + ab$. Prove that $(G,*)$ is a group.

b) Let G be a group in which $(ab)^n = a^n b^n$ for three consecutive integers

and for all $a, b \in G$. Prove that $(G,\cdot)$ is an abelian group.

c) Let $(G,\cdot)$ be a group and $a, b \in G$. Prove that

order of $a =$ order of $b^{-1}ab$.

$4 + 4 + 3 = 11$

Q.7.a) Define left coset. If H be a subgroup of G, prove that any two left cosets are either identical or disjoint.

b) Define index of a subgroup in a group. Find the index of $\{0,3,6,9\}$ in

$(Z_{12}, \oplus)$.

$2 + 4 + (2 + 3) = 11$

Q.8.a) State and prove of Lagrange's theorem on a finite group.

b) Prove that the intersection of two normal subgroups of a group G is a normal subgroup of G.

$(2 + 4) + 5 = 11$

Q.9.a) Let G be a group. Show that $f: G \to G$ given by $f(x) = x^{-1}$ is an

isomorphism $\Leftrightarrow$ G is abelian.

b) Let R be a ring and $a, b \in R$. Prove that

$$(-a)(-b) = ab$$

$$a(b - c) = ab - ac$$

$6 + (3 + 2) = 11$

Q.10.a) (i) For a graph $G = (V, E)$ what is the largest possible value for $n(V)$ if $n(E) = 19$ and $deg(v_i) \geq 4$ for all $v_i \in V$ ?

(ii) Prove that the number of edges in a bipartite group with n vertices is at most $\frac{n^2}{2}$ .

b) If G is a connected simple planar group with $n(\geq 3)$ vertices $m(> 2)$ edges and r regions. Prove that

(i) $m \geq \frac{3r}{2}$

(ii) $m \leq 3n - 6$

(iii) Further if G is triangle -free then $m \leq 2n - 4$

$(3 + 2) + (2 + 2 + 2) = 11$