

BE CST 6th Semester End-Term Examination April '2010

Department of Computer Science and Technology

Theory of Computation (CS - 002)

•F.M.: 70

TIME : 3 hrs

- Attempt question no. 1 and any five from the rest.
 - Answers should **be** in your own words as far as practicable.
 - Make your own assumptions as and when necessary and state them at proper places.
1. Write short notes on any three from the following.
 - (a) Universal Turing Machine.
 - (b) Primitive recursive functions from strings to string.
 - (c) Grammar to compute functions.
 - (d) Parse tree as representation of derivations under a grammar. III]
 2. Many authors allow a Turing machine both to move the head and to write a symbol at the same time. For them, a Turing machine is a quadruple $(A^*, L, \langle \rangle, s)$, where A^* , L and $\langle \rangle$ have their conventional meaning, but s is a function from $A^* \times \{K \cup \{/\ast\}\} \times S \rightarrow \{L, R, S\}$ (S meaning "stay in the place").
 - (a) Define carefully the relation H between configurations for these more general machines.
 - (b) Explain precisely how to convert one of these machines into a Turing machine of the standard type, and vice versa.
 3. A language L is definite if there is some k such that, for any string u , whether $u \in L$ depends only on the last k symbols of u .
 - (a) Rewrite the definition (for definite language) more formally.
 - (b) Show that every definite language is accepted by a finite automaton.
 - (c) Show that the class of definite languages is closed under union and complementation. [4*3]
 4. Show that for any deterministic finite automaton $M = (A^*, S, S, s, F)$, A^* accepts an infinite language if and only if M accepts some string of length greater than or equal to $|A^*|$ and less than $2|A^*|$. [12]
 5. For each of the following languages construct a Pushdown Automaton that accept the language.
 - (a) $\{ \langle^n b^m \mid m < n < 2m \rangle \}$
 - (b) $\{ a^i b^j \mid i = j \}$
 - (c) $\{ a^i b^j \mid i + j = 2A \}$ [4*3]

G. For each of the following languages construct a grammar that generates the language.

(a) $\{0^n r ; n > 0, r \in \{ft, ft\}^* \text{ and } \langle \langle \rangle\}$

(b) $\{a^m r^* \mid i \leq A^i > 0 \text{ and } / \wedge j \text{ or } j / A\}$

(c) $\{ / \dots + \dots \} , i, m > 0 \}$ [4+3]

7. Show from definition that the following functions are primitive recursive.

(a) $A^f : \mathbb{N}^* \rightarrow \mathbb{N} \text{ At } j > 0, A^f(m, n, \dots, n^*) = j$

(b) $f : \mathbb{N}^+ \rightarrow \mathbb{N}^+, f(m, n) = \lfloor 2 \cdot \min(m, n) \rfloor$. f is primitive recursive

(c) $sg : \mathbb{N} \rightarrow \{0, 1\}$. $sg(u) = 1$ if $u = 0$ and $sg(u) = 0$ otherwise [5+4+3]

8. (a) Propose an algorithm that, generates a Pushdown Automaton $M = (A, \Sigma, \Gamma, \delta, q_0, F)$ for a given Context Free Grammar $G = (V, \Sigma, P, S)$, such that the language generated by G is same as the language accepted by M .

(b) Let $G = (V, \Sigma, P, S)$ be a grammar with $K = \{S, n, fc, r\}$, $\Sigma = \{a, b, r\}$, $R = \{S \rightarrow tiStt, S \rightarrow ftSfr, S \rightarrow c\}$. Following the algorithm you have proposed in the previous step, construct a Pushdown Automaton accepting the language $L[G]$. [9+3]

U. (a) "Subsets of regular languages are not always regular," - true or false? Justify your answer.

(h) Let L be a context-free language and R be a regular language. Is $L \cdot R$ necessarily context-free? What about $R \cdot L$? Formally justify your answer. [4+4]