B.E. (CST) Part-II 4th Semester Examination, 2010

Discrete Structure
(CS-401)

Time : 3 hours Full Marks : 70

Use separate answerscript for each half.

FIRST HALF

(Answer any FIVE questions.)

1. a) For any two sets A and B prove that $(A \cap B)' = A' \cup B'$.

   b) Let $f: R \to R'$ be defined by $f(x) = e^x$, $x \in R$. Prove that f is invertible and find f^{-1}. [3+4]

2. a) Let p be an equivalence relation on a set S and a, b ∈ S. Prove that the equivalence classes cl(a) and cl(b) are either equal or disjoint,

   b) Find the equivalence classes determined by the equivalence relation p on Z defined by "a p b if and only if a - b is divisible by 5" for a, b ∈ Z. [4+3]

3. a) Find out the particular solution of the difference equation: $a_r - 2a_{r-1} = 3.2^r$.

   b) Show that for any discrete numeric function a, $S^{-1}(\nabla a) = \Delta a$, where S, ∇ and Δ has their usual meanings. [4+3]

4. a) How many different reflexive relations can be defined on a set A containing N elements?

   b) Compute the generating function of the discrete numeric function $(0, 1^2, 2^2, \ldots, r^2, \ldots)$ and following it evaluate the sum $1^2 + 2^2 + 3^2 + \ldots + r^2$, [2+5]

5. a) Let a and b are two discrete numeric functions such that $a_r = 3^r$; $r \geq 0$ and $b_r = 5^r$; $r \geq 0$. Compute a*b where * is the convolution of two numeric functions.

   b) Solve the difference equation

   $a_r - r a_{r-1} = r!$ for r> 1, given that $a_0 = 2$. [3+4]

6. a) Prove, using the concept of generating function that the number of ways of selecting r objects from n objects with unlimited repetitions is C(n + r - 1, r)

   b) Let a_r be the number of subsets of the set {1, 2, 3, ... , r} that do not contain two consecutive numbers. Determine a_r. [3+4]

SECOND HALF

(Answer anu THREE from the rest.
Va marks are reserved for neatness and clarity.)

7. a) State Second Principle of Mathematical Induction. How does if differ from First Principle of Mathematical Induction?

   b) Prove that a straight fence with n fence posts has n-1 sections for any n> I using both First and Second Principle of Mathematical Induction.

   c) Prove that for all n^ 10, $2^n > n^3$.

   d) Prove that for all integer n(n>0),
   $0.1 + 1.2 + 2.3 + ... + n(n + 1) = \frac{n(n+l)(n + 2)}{\wedge}$• I(l+K)+(2V$x2)+2+3|

8. a) Prove that P -» Q = (P A~ Q) -» •.
   What conclusion can you make from the above on Proof by contradiction?

   b) Use Proof by contradiction to prove the following :
   If n is an integer, then $n^2 + 2$ is not divisible by 4.

   c) What is Proof by cases?

   d) Let A, B and C be three inhabitants of an island where there are either "knights" or "knaves". The "knights" always tell truth and the "knaves" never tell truth. Two individuals are of same type if they are both knights or both knaves. Suppose A says, "B is a knave" and B says "A and C are of the same type". Use proof by cases to establish that C is a knave.

   e) Prove that if n+1 separate passwords are issued to n students, the some students gets n>2 passwords. |(l+W)+3+1+3+31

9. a) For each of the following formulas, determine whether it is valid, invalid, inconsistent, consistent or some combination of these.
   (0 (P->Q)->(~Q->~P)
   (ii (P _> Q) (Q _> P)

   b) Given that if the congress refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the president of the firm resigns. Prove that strike will not be over if the congress refuses to act and the strike just starts. [(2V4x2)+6Wl

10. a) Convert the following into FOPL coif choosing appropriate predicates
    (i) Every rational number is a real number.

(ii) There exists a number that is prime.

(iii) For every number there is one and only one immediate successor.

(iv) There is no number for which 0 is the immediate successor.

(v) For every number other than 0 there is one and only one immediate predecessor.

Write an algorithm to convert a coff in FOPL into Prenex Normal Form.

|(JxS)+6ViJ

For the following interpretation (D = {a, b}),

P(a,a)	P(a,b)	P(b,a)	P(b, b)
T	F	F	T

determine the truth values of the following formulas.

(i) 3xVyP(x,y)

(ii) VxVy(P(x,y)->P(y,x))

(iii) VxP(x,x).

b) All numbers divisible by 2 are even numbers. Number 25 is not an even number. Conclude that number 25 is not divisible by 2 (apply FOPL Proof).

I(2x3)+5!4|