BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR
B.E. 4th SEMESTER FINAL EXAMINATIONS, 2013
Mathematics- IV-1
(MA-401/1)

Full Marks: 70 **Time: 3 hrs**
Branch : ETC & CST

Use separate answer script for each half

First Half

Answer as much as you can or wish to in this half. The maximum you can score in this half is 35.

1.a.) Prove that for <u>any</u> positive integer n, there exists an <u>abelian</u> group of order n.

b.) Suppose G is a <u>finite</u> group and H is a subgroup of G. Prove that the order of H is a divisor of the order of G.

[4+7=11]

2.a.) Define a <u>normal</u> subgroup. If N is a <u>normal</u> subgroup of a group G, prove that the collection of right cosets of N in G form a group. Also establish that if G is abelian, so is the group thus formed by the right cosets of N in G.

b.) Suppose G is a group and $a \in G$. The *normalizer* of the element a consists of all elements of G which <u>commute</u> with the element a. Show that the *normalize* of a, denoted by $N(a)$, is, in fact, a subgroup of G.

c.) If G is a group and H is a subgroup of G, define the *index* of H in G.
Show, by an example, that the *index* of H in G maybe <u>finite</u> even when both G and H are of infinite order.

[5+4+5=14]

3.a.) Suppose ϕ is a *homomorphism* of a group G <u>onto</u> a group $\overline{G}$ with *kernel* K.
Show that G/K is *isomorphic* to $\overline{G}$.

b.) Prove that the *steady state distribution* of a $M/M/\infty$ queueing system with constant arrival and service rates is Poisson.

[7+4=11]

4. Following questions may have <u>one or more than one correct option</u>. Answer <u>any three with justifications.</u>

(i) The set G={1, -1, I, -i } is a group under multiplication

a) The order of G is 4.

b) I is the generator of G.

c) 1 is the identity element of G.

d) There is no identity element.

(ii). If I is the additive group of integers and E the additive group of even integers, the mapping $f: I \to E$ is given by $f(x) = 2x$ where $x \in I$.

a) f is one-one.

b) f is onto.

c) f preserves the group composition.

d) f is isomorphism.

(iii). Given permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$

a) Order of permutation p is 1.

b) Order of permutation p is 2.

c) Order of permutation p is 3.

d) Order of permutation p is 4.

(iv). In a group G, $a^5 = e,\ aba^{-1} = b^2$ for $a, b \in G$ then

a) $b^4 = a^2ba^{-2}$

b) $b^{16} = a^6ba^{-6}$

c) $o(b) = 31\ , b \neq e$

d) $o(b) = 32\ , b \neq e$

[3 X 3 = 9]

5.a). If m and M denote the minimum and maximum degrees of the vertices of a graph with n_v vertices and n_e edges. Show that $m \leq \frac{2n_e}{n_v} \leq M$.

b) If (R, +, ·) is a ring, prove that

(i) a.0 = 0 = 0.a $\forall a \in R$

(ii) a.(- b) = (-a). b = -(a.b) $\forall a, b \in R$

(iii) (-a). (-b) = a.b $\forall a, b \in R$

[3+6 = 9]

6.(i). Patients arrive at a government hospital for emergency service at the rate of one every hour. Currently only one emergency can be handled at a time. Patients spend an average of 20 minutes receiving emergency care. The doctor wishes to have enough seats in the waiting room so that no more than about 1% of arriving patients will have to stand. Find

(a) the probabilities that a patients arriving at the hospital will have to wait.

(b) the average time a patient spends in the queue that forms.

(c) average time a patient spends in the queue.

(d) average time a patient spends in the system.

(e) probability that there will be five or more patients waiting for the service.

(ii) In queueing model (M/M/1: ∞/ FCFS) , establish average numbers of customers in the system and find the probability that the numbers of customers in the system is greater than or equal to k in terms of traffic intensity.

[10+7 = 17]

SECOND HALF

Answer any **THREE** questions

Two marks are reserved for general proficiency

7. Using the method of least squares, fit a polynomial of the second degree to the following data:

x	0.0	1.0	2.0
y	1.0	6.0	17.0

after deducing the associated formula. 11

8. Evaluate

$$I=\int_0^{1.2} \frac{1}{1+x}dx,$$

by using Romberg's method, correct to three decimal places, after deducing the associated formula. 11

9. a) Using Runge-Kutta method of fourth order, find y (1.1) correct to five decimal places, given that $y' = x^2+y^2$; $y(1)=0$, where the symbol (') denotes the derivative with respect to x. 6

b) Deduce Milne's or any other predictor and corrector formula to solve the first order differential equation $y'= f(x, y)$ with the initial condition $y(x_0)$. 5

10. a) Explain the following terms in the Calculus of Variations :

i) Functional, ii) Brachistochrone problem 4

b) On what curves can the functional

$$J[y(x)] = \int_1^2 (y'^2 - 2xy)dx; \quad y(1) = 0, y(2) = -1$$

attain an extremum? 7

11. a) Determine the plane curve of quickest descent as a particle moving on a smooth surface falls under gravity from a fixed point A(0,0) to another fixed point B(a, b). 7

b) Find the extremals of the functional

$$J[y(x)] = \int_1^3 (3x - y)y dx$$

that satisfy the boundary conditions $y(1)= 1$, $y(3) =9/2$.. 4