# B.E. 3rd Semester Final Examination, 2011
# Mathematics-III B (MA-302)

**Full Marks- 70** **Branch-EE/ETC/IT**

**Time – 3 hours**

Use a separate answer script for each half.

**FIRST HALF**

Answer <u>any three</u> questions in this half. 2 marks are reserved for general proficiency in this half.

1. (a) Ram has three coins in his pocket, two fair ones but the third biased with probability of heads p and tails 1- p. One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins?

   (b) Find the *moment generating function* (m.g.f.) of a N ($\mu$ , $\sigma^2$ ) random variable. If X and Y are *independent* standard normal random variables, find the distribution of X+Y.

   6+5=11

2. (a) State and prove the theorem of total probability.

   (b) 29 balls are distributed in 9 boxes <u>at random</u>. Each box is capable of accommodating all 29 balls. Find the variance of the number of empty boxes.

   5+6=11

3. (a) X and Y are <u>independent</u> random variables with mean 2 and variance 3. Find the correlation coefficient between 5X+Y and X-2Y. Is this same as the correlation coefficient between X and Y ? Justify your answer.

   (b) $X_1, X_2, ..... X_8$ are *independent and identically distributed* (i.i.d.) random variables each having an exponential distribution with parameter 5. Derive the probability distribution of $\min.(X_1, \ldots\ldots, X_8)$.

5+6=11

4.(a) Derive from first principles the *normal equations* for a simple regression model with one independent variable. Solve these equations to obtain the *least square estimators* of the two parameters in the model.

(b) Derive the maximum likelihood estimators of the parameters $\mu$ and $\sigma^2$ given a random sample of size 14 from a normal population.

6+5=11

5. (a) X is a random variable such that $E(X) = 12$ and $E(X^2) = 144$. Find $E(X^{144})$.

(b) Prove that there *does not exist any random variable* whose first four moments equal the first four natural numbers respectively.

3+8=11

## SECOND HALF

### Answer any Five $7 \times 5 = 35$

6. If $\lambda \Delta t + 0(\Delta t)$ is the probability of a single arrival during a small interval of time $\Delta t$ and if the probability of more than one arrival is negligible, prove that the arrival follows the poisson's law. Hence obtain the distribution of time interval between two consecutive arrivals.

7. Under certain assumptions to be stated by you and with usual notations establish the following difference equations in steady state situation of (M/M/1): (N/FIFO) queueing model

$$\mu p_1 = p_0 \lambda, \mu p_{n+1} = (\lambda + \mu) p_n - \lambda p_{n-1} \text{ when } 1 \leq n \leq N-1$$

$$\mu p_N = \lambda p_{N-1}$$

Hence show that $p_n = \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}$ where $\rho = \dfrac{\lambda}{\mu} \neq 1,\ 0 \leq n \leq N$

8. A road transport company has one reservation clerk on duty at a time. He handles information of bus schedules and makes reservations. Customers arrive at a rate of 8 per hour and the clerk can service 12 customers on an average per hour.

(i) What is the average number of customers waiting for the service of the clerk in the system and also in the queue.

(ii) The management is contemplating to install a computer system to handle the information and reservations. This is expected to reduce the service time from 5 to 3 minutes. The additional cost of having the new system works out to be Rs. 50 per day. If the cost of goodwill of having to wait is estimated to be 12 paise per minute spent waiting before being served. Should the company install the computer system? (Assume 8 hours working day)

9. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait. While other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an verage of 12 per hour. Assuming poission arrival and exponential service distribution, Find the steady state probabilities for various number of trains in the system.

10. The owner of Metro Sports wishes to determine how many advertisements to place in the selected three monthly magazines A, B and C. His objective is to advertise in such a way that total exposure to principal buyers of expensive sports good is maximized. Percentage of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements placed multiplied by the number of principal buyers. The following data may be used:

| | MAGAZINE | | |
|---|---|---|---|
| | A | B | C |
| Reader | 1 Lakh | 0.6 Lakh | 0.4 Lakh |
| Principal buyer | 20% | 15% | 8% |
| Cost of Adv. | 8000 | 6000 | 5000 |

The budgeted amount is at most Rs. 1 Lakh for the advertisements. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each have at least 80 advertisements. Formulate this as a linear programming model.

11. Consider the $\ell.p.p$

Maximize $Z = \underset{-}{c}\,\underset{-}{x}$

Subject to $A\,\underset{-}{x} = \underset{-}{b}$

$\underset{-}{x} \geq \underset{-}{0}$

With reference to the above $\ell.p.p$ prove or disprove the followings

(i) The set of all feasible solutions is a convex set.

(ii) If S be the set of all feasible solutions and if $\underset{-}{x}^* \in S$ maximize the objective function then $\underset{-}{x}^*$ also minimizes the function $\bar{Z} = (-\underset{-}{c})\underset{-}{x}$ over S.

12. Use Big-M method to solve:

Maximize $Z = 6x_1 + 4x_2$

Subject to

$2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

Is the solution unique? Justify in support of your answer.