B.E.(CE/ME/MET/MIN) 3rd Semester Final Examination, December 2011

Mathematics-IIIA (MA-301)

Time : 3 hours Full Marks : 100

Use separate answer script for each half.

FIRST HALF

Answer Q.No.1 and any two from the rest.

1. **Answer any three questions.** **(3x5)**

(a) Define analyticity and singularity of a complex function at a point. Give an example (with proper explanation) of a function which is continuous everywhere, nowhere analytic and has no singular point.

(b) Define harmonic conjugate. Show that if two functions u(x,y) and v(x,y) are to be harmonic conjugates of each other, then both u and v must be constant functions.

(c) State Cauchy-Goursat theorem. Give an example to show that the condition of Cauchy-Goursat theorem are sufficient but not necessary.

(d) Evaluate the line integral

$$\int_0^{1+i} f(z)dz \text{ where } f(z) = y-x-3ix^2$$

(i)along the straight line from z=0 to z=1+i and (ii) along the imaginary axis from z= 0 to z=i (i.e., OA) and then along a straight line parallel to the real axis from z=i to z=1+i (i.e., AB).

Hence determine the value of the contour integral

$$\int_\Gamma f(z)dz \text{ where } \Gamma \text{ is the closed contour OABO.}$$

What conclusion can be drawn regarding analyticity of f(z).

(e) Determine all singular points, their nature and residue at the singular points of the following functions:

$$(i)\ \frac{1-\cosh z}{z^3} \quad \text{(ii) } \cot z\ .$$

2. (a) State and prove Cauchy Integral formula.

(b) Evaluate the contour integral using Cauchy Integral formula

$$\int_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz \text{ where c is the circle } |z|=3 \text{ described in the positive sense.}$$

(6+4)

3. (a) If f(z) is analytic in an open region R and $z_0 \in R$ then prove that f(z) can be expanded in the form

$$f(z) = \sum_0^\infty a_n (z - z_0)^n \text{ where } a_n = \frac{f^{(n)}(z_0)}{n!}, \ n=0,1,2,\ldots\ldots\ldots .$$

Specify the domain in which expansion is valid.

(b) Represent the function $f(z) = \frac{z+1}{z-1}$ by (a) its Maclaurin series and give the region of validity for the representation , (b) its Laurent series for the region $1 < |z| < \infty$.

(6+4)

4. Apply Cauchy residue theorem to evaluate the following integrals (any two) :
(i) $\int_0^{2\pi} \frac{dt}{(a+b\cos t)^2}$ ($a > b > 0$) , (ii) $\int_0^\infty \frac{dx}{x^4+1}$, (iii) $\int_0^\infty \frac{x \sin x\, dx}{x^2+a^2}$ (Re(a)> 0) .

(5+5)

5. (a) Let C be the line segment from z=i to z=1. Without evaluating the integral show that

$$\left|\int_C \frac{dz}{z^4}\right| \le 4\sqrt{2} .$$

(b) State Dirichlet's conditions for convergence of a Fourier series. Prove that the even function $f(x) = |x|$ in $-\pi < x < \pi$ has a cosine series in Fourier's form as

$$|x| \sim \frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\ldots\ldots\ldots \right].$$

Apply Dirichlet's conditions of convergence to show that the series converges to $|x|$ throughout $-\pi \le x \le \pi$.

Also show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}$.

(3+7)

Second Half

Two marks are reserved for general proficiency.

Answer any three questions

6. a) A tailor has 80 sq m of cotton material and 120 sq m of woolen material. A suit requires 1 sq m of cotton and 3 sq m of woolen material and a dress requires 2 sq m of each. A suit sells for Rs 500 and a dress sells for Rs 400. Formulate a LPP in terms of maximizing the income.

   b) Solve graphically the LPP.

   Minimize $z = 2x_1 + 3x_2$

   Subject to

   $$2x_1 + 7x_2 \geq 22$$
   $$x_1 + x_2 \geq 6$$
   $$5x_1 + x_2 \geq 10, \quad x_1, x_2 \geq 0$$

   5+6 = 11

7. a) Show that $x_1 = 5, x_2 = 0, x_3 = -1$ is a basic solution of the system of equations

   $$x_1 + 2x_2 + x_3 = 4$$
   $$2x_1 + x_2 + 5x_3 = 5$$

   Find the other basic solutions if there be any.

   b) Show that Hyperplane is a convex set.

   7+4 = 11

8. a) Prove that in E^2, the set $\{(x,y) \mid x + 2y \leq 5\}$ is a convex set.

   b) Show that the LPP

   Maximize $z = 4x_1 + 14x_2$

   Subject to

   $$2x_1 + 7x_2 \leq 21$$
   $$7x_1 + 2x_2 \leq 21$$
   $$x_1, x_2 \geq 0$$

   admits of an infinite number of solutions.

   4+7 = 11

9. a) Show that the set of all feasible solutions of a LPP is a convex set.

   b) Solve the following LPP

   Maximize $z = 2x_1 + 3x_2 + x_3$

   Subject to

   $$-3x_1 + 2x_2 + 3x_3 = 8$$
   $$-3x_1 + 4x_2 + 2x_3 = 7$$
   and $x_1, x_2, x_3 \geq 0$

   4+7= 11

10. Solve the boundary value problem $C^2u_{xx} = u_t$ with the conditions $u(l,t) = 0$ for all for all $t \geq 0$, $\frac{\delta u}{\delta t}(0,t) = 0$ and $u(x,0) = 20x$ for $0 < x < l$

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