Bengal Engineering and Science University, Shibpur

BE (EE/ETC/CST/IT) 3rd Semester Final Examination , 2013

Mathematics-III-1

(MA-301/1)

(Use separate answer scripts for each half)

Time : 3 hours   Full Marks : 70

First Half

Answer any five questions (Marks: 7X5=35)

1.

(a) If $A_1, A_2, \cdots A_n$ be any n events connected to a random experiment E then establish the following inequality

$$p(A_1, A_2, \cdots A_n) \geq \sum_{i=1}^{n} p(A_i) - (n-1)$$

b) Consider two events A and B such that P(A)=1/4 and P(B/A)=1/2, P(A/B)=1/4. With proper justification verify whether the following is true or false?

(i) $P(\overline{A}/B) = 3/4$. (ii) $P(A/B) + P(A/\overline{B}) = 1$

where $\overline{A}$ is the complementary event of A.

2. (a) Show that in defining mutual independence of n events $A_1, A_2, \cdots A_n (n > 2), 2^n - n - 1$ relation are required.

(b) Suppose that there is a chance for newly constructed house to collapse whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the house collapse if the design is faulty is 95% and otherwise it is 45%. It is seen that the house collapsed. What is the probability that it is due to faulty design?

3. a) State 'weak law of large numbers'. Let X_i assume the values i and $-i$ with equal propabilities. Show that the weak law of large numbers cannot be applied to the independent variables $X_1, X_2, X_3 \cdots\cdots$

(b) The p.d.f of a continuous random variable X is

$f(x) = y_0 e^{-|x|}, \quad -\infty < x < \infty.$

Show that (i) $y_0 = 1/2$, (ii) Mean=0 and (iii) Standard deviation $= \sqrt{2}$.

4. a) Let X_1, X_2, X_3 is random sample of size three drawn from a population with mean μ and variance σ^2. Let $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_2 - 4X_3$, $T_3 = (\lambda X_1 + X_2 + X_3)/3$.

(i) Are T_1, T_2 unbiased estimators of μ?

(ii) Find λ for which T_3 is an unbiased estimators of μ.

b) From a random sample of size n drawn from normal population $N(\mu,\sigma^2)$, find the maximum likelihood estimator for μ when σ^2 is known.

5. a) For geometric distribution $p(x)=2^{-x}$, $x=1, 2, 3,\ldots\ldots$ Prove that Chebyshev's inequality gives $P(|X-2|\le 2)\ge 1/2$, while the actual probability is 15/16.

b) The daily consumption of milk in excess of 20,000 liters in a town is approximately exponentially distributed with parameter 1/3000. The town has a daily stock of 35000 liters. What is the probability that of two days selected at random the stock is insufficient for both days?

6.a) With usual notations show that the angle between the two regression lines is given by $\tan^{-1}\left\{\frac{1-\rho^2}{\rho}\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)\right\}$. What happens when the correlation coefficient $\rho=\pm 1$?

b) Suppose $X_1, X_2,\ldots\ldots, X_n$ is a random sample of size n drawn from a population P having finite mean μ and finite variance σ^2. Prove that $E\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2\right)=\sigma^2$, where $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$.

7.a) If t is any positive real number show that the function $p(x)=e^{-t}(1-e^{-t})^{x-1}, x=1,2,\ldots..$ can represent a probability mass function of a random variable X assuming values $1,2,\ldots\ldots$ Hence find E(x).

b) If $X\sim B(3,1/3)$ and $Y\sim B(5,1/3)$ are two independent binomial variates find $P(X+Y\ge 1)$.

8.a) One per thousand of a population is subject to certain kinds of accident each year. Given that an Insurance Company has insured five thousand persons from the populations. Find the probability that at most two persons will incur this accident.

b) At JEE each candidate is admitted or rejected according to whether he has passed or failed in that test. Of the candidates who are really capable 80% pass the test and of the incapable 20% pass the test. Given that 40% of the candidates are really capable, find the proportion of capable candidates.

SECOND HALF

Answer any **three** questions

Two marks are reserved for general proficiency

9. a) Solve the boundary value problem

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2} \quad , o < x < 3 \text{ and } t > 0,$$

$$u(0,t) = 10, u(3,t) = 40, u(x,0) = 25$$

$$\text{and } u \text{ is bounded for } o < x < 3 \text{ and } t > 0.$$

b) If C be a constant, then show that, in $0<x<\pi$,

$$C = \frac{4C}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$$

8+3

10. a) State and prove convolution theorem for Fourier transform.

b) Find Fourier transform of the function $H(a-|x|)$, where H denotes Heaviside unit step function. 7+4

11. a) State Cauchy- Goursat Theorem.

Evaluate $\frac{1}{2\pi i}\oint_C \frac{e^z}{z-2}dz$, if C is the circle $|z|=1$.

b) Evaluate $\oint_C (x^2 + iy^2)ds$ around the circle $|z|=2$, where s is the arc- length.

c) Let f(z) be analytic in a region R bounded by two simple closed curves C_1 and C_2 and also on C_1 and C_2. Prove that

$$\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz,$$

where C_1 and C_2 are both traversed in the counter- clockwise direction.

(1+2)+4+4

12. a) State and prove Cauchy's integral formula.

b) Evaluate $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{(z^2+1)^2}dz$, if $t > 0$ and C is the circle $|z| = 3$.

c) Determine the singularities and their nature of the following functions:

(i) $\frac{z}{(z+1)(z-1)^2}$, (ii) $e^{\frac{1}{z}}$. (1+3)+4+(2+1)

13. a) State and prove Cauchy's residue theorem.

b) Use method of contour integration to prove that

$$\int_0^{2\pi} \frac{dt}{1+a^2-2a\cos t} = \frac{2\pi}{1-a^2}, (0 < a < 1).$$

c) Find the Taylor's and Laurent's series which represent the function

$$f(z) = \frac{z^2-1}{(z+2)(z+3)}$$

(i) when $|z| < 2$, (ii) when $2 < |z| < 3$. (1+3)+4+(1+2)