B. E. (All Branches) 2nd Semester Final Examination, April 2013

Mathematics II (MA 201)

Time: 3 hours   Full Marks: 70

(Use separate answer script for each half)

First Half

(Answer any THREE questions. Two marks are reserved for general proficiency)

1a) Prove that the improper integral $I = \int_a^b \frac{dx}{(x-a)^n}$ converges if and only if $n < 1$.

b) Test the convergence of $\int_1^{\infty} \frac{\cos x}{\sqrt{1+x^3}}\, dx$.   6 + 5

2 a) Using Mean Value Theorem of integral calculus show that

$$\frac{1}{4} < \int_0^{\frac{1}{4}} \frac{dx}{\sqrt{1-x^4}} < \frac{1}{\sqrt{15}}$$

b) Prove that $\Gamma(n+1) = n\Gamma(n)$ where $n > 0$.

c) Using the above recurrence relation show that

$$\int_0^{\infty} e^{-x^2} x^9\, dx = 12$$

5+ 3 + 3

3 a) Let $V = \{(x, y) : x, y \in F\}$ and F be the field of real numbers. Show that V is not a vector space under addition and scalar multiplication defined as

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$a(x, y) = (o, ay)$$

b) Prove that the vectors (1, 1, 1), (1, 1, 0) and (1, 0, 0) form a basis in R^3. Prove also that the vector (1, 3, 1) can replace any one of the three vectors of the basis to form a new basis.

<u>4 + 7</u>

4 a) Let $S = \{(x, y, z) \in R^3 : 3x - y + z = 0\}$. Show that S is a subspace of R^3. Find a basis for S.

b) Prove that the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear if and only if the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ be less than three.

<u>7 + 4</u>

5 a) Show that the value of the determinant of an orthogonal matrix is either +1 or -1.

b) Investigate for what values of λ and μ the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

has i) no solution ii)an unique solution iii) an infinite number of solutions.

<u>4 + 7</u>

6 a) Define eigen value and eigen vector of a square matrix.

b) Find eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

<u>4 + 7</u>

SECOND HALF

(Answer any THREE questions. Two marks are reserved for general proficiency)

7. Solve the following differential equations:

a) $\frac{d^2y}{dx^2}$ -y = coshx

b) $\frac{d^2y}{dx^2}$ + 4y = sin2x

c) $x^2\frac{d^2y}{dx^2}$ -4x $\frac{dy}{dx}$ +6y= x. (4+3+4)

8. a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+a^2$y = sec ax.

b) Solve: $\frac{d^2y}{dx^2}-\frac{dy}{dx}$ -2y= sin2x. (8+3)

9. a) Solve the differential equation

$$\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+3y=0$$

in series about the ordinary point x=0. (7+4)

b) Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$, where $J_n(x)$ is the Bessel function of first kind of order n.

10. a) Find the Laplace transform of the function cos at.

b) Let the Laplace transform of the function f(t) be denoted by L{f(t)} or F(s). Prove that

$$L\{f(at)\} = \frac{1}{a}F(\frac{s}{a}), \text{ where } a>0.$$

c) Find the Laplace transform of the function f(t), where

f(t)=1, if $0 < t < 2$

=$2t^2$, if $t > 2$. (3+3+5)

11. a) If f(t) be continuous and L{f(t)} = F(s), then prove that L{$f'(t)$} = - f(0) + s F(s).

Apply this result to show that L{$f''(t)$} = $-f'(0) - s f(0) + s^2 F(s)$.

b) Solve the following differential equation using Laplace transform:

(D^2-3D+2) y= 4 e^{2t} with y(0)= -3 and $y'(0)$= 5 where $D \equiv \frac{d}{dx}$. (5+ 6)