B.E. 1st Semester Examination-2013

Mathematics- I

MA-101

Full Marks-70 **Time-3 hrs.**

Use separate answer script for each half

First half

Answer any **three** questions
Two marks are reserved for general proficiency

1. a) State and prove the Leibnitz theorem for differentiation of the n-th order of product of two functions.

   b) If x+y = 1, prove that the n-th order derivative of $x^n y^n$ is given by
   $n!\{y^n - ({}^nC_1)^2 y^{n-1} x + ({}^nC_2)^2 y^{n-2} x^2 + \ldots\ldots\ldots + (-1)^n x^n\}$

   (1+5+5)

2. a) Verify Rolle's theorem for the function $f(x)=|x|$ in $|x| \leq 1$

   b) Expand the function log (1+x) in powers of x in infinite series stating the condition under which the expansion is valid.

   (4+7)

3. a) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) + \cot u = 0$.

   b) If $u = x f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ then show that $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = 0$.

   (5+6)

4. a) Define asymptote of a curve. Prove that the asymptotes of the cubic $(x^2 - y^2)\, y - 2ay^2 + 5x - 7 = 0$ form a triangle of area a^2.

b) Find the point on the parabola $x^2 = 2y$ nearest to the point (0, 3) using the method of Lagrange's multiplier.

(1+5+5)

5. a) If (α, β) be the co-ordinates of the centre of curvature of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$, at (x, y), then show that $\alpha+\beta = 3(x+y)$.

b) Define curvature of a curve at a point P(x, y). Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point where $y = x$ cuts it.

(5+1+5)

SECOND HALF

Answer question no. 1 and any two from the rest.

1. **Answer any three questions.** (3x5)

(a) Find the n-th partial sum of the infinite series

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\text{..................to }\infty.$$

Hence find the sum of the series if it is convergent.

(b) Prove that $\lim_{n\to\infty} u_n = 0$ if the infinite series $\sum_1^\infty u_n$ is convergent. Is the converse true ? Give an example in support of your answer.

(c) Determine the volume of a sphere of radius a with the help of a triple integral.

(d) Prove that

(i) Curl grad f = $\vec{0}$, and (ii) div curl $\vec{F} = 0$.

2. (a) Examine the convergence of the infinite series $\sum_1^\infty \frac{n^n}{n!}$. (5x2)

(b) Evaluate the double integral

$$\iint_R (x^2+y^2)\,dx\,dy$$

where R is the region enclosed by the triangle with vertices at the points (0,0), (1,0) and (1,1).

3. (a) Find the interval of convergence of the power series (5x2)

$$x+\frac{x^3}{2.3}+\frac{1.3.x^5}{2.4.5}+\frac{1.3.5.x^7}{2.4.6.7}+\text{.................to }\infty.$$

(b) Evaluate the double integral by transforming into polar coordinates

$$\iint_R \sqrt{4a^2-x^2-y^2}\,dx\,dy$$

where R is the upper half of the circle

$$x^2+y^2-2ax=0.$$

4. (a) Apply vector method to determine the equations of the tangent plane and normal line to the surface xyz = 4 at the point (1,2,2).

(b) Evaluate the line integral using Green's theorem in plane

$$\oint_{\Gamma} \{(\cos x \sin y - xy)dx + \sin x \cos y \, dy\}$$

where Γ is the circle $x^2 + y^2 = 1$ in the xy-plane described in the positive sense. (5x2)