B.E. 1st SEMESTER EXAMINATIONS, 2011

Mathematics- I

(MA-101)

Full Marks: 70 Time: 3 hrs

Use separate answer script for each half

First half

Answer question no. 1 and any two from the rest of this half

1. **Answer any five (5x3=15)**

a) If $f(x) = \tan x$, then prove that

$$f^n(0) - {}^nC_2 f^{n-2}(0) + {}^nC_4 f^{n-4}(0) - \ldots\ldots\ldots\ldots = \sin(\frac{n\pi}{2}).$$

b) If $z = f(x, y)$, where $x = e^u \cos v,\ y = e^u \sin v$, show that

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}.$$

c) If $u = x\phi(y/x) + \psi(y/x)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\phi(y/x)$.

d) Expand $3x^3 + 2x^2 - 7$ in powers of $x+3$.

e) Determine the radius of curvature of the curve $x^2 + y^2 - 4x + 5$ at the point (5, 0).

f) Determine the asymptotes to the curve $x^4 - y^4 + 3x^2y + 3xy^2 + xy = 0$.

g) Determine the Cauchy's remainder after n terms in the expansion of log(1+x).

2. a) State and prove Taylor's theorem with Lagrange's form of remainder.

b) Prove that $Lt \dfrac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$, provided $f''(x)$ is continuous. (10)

3. a) If H is a homogeneous function of degree n in x *and* y and if $u = (x^2 + y^2)^{-\frac{n}{2}}$,

then prove that $\dfrac{\partial}{\partial x}\left(H\dfrac{\partial u}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(H\dfrac{\partial u}{\partial y}\right) = 0$.

b) If $V = (x^2 + y^2 + z^2)^{1/2}$, then show that $V_{xx} + V_{yy} + V_{zz} = 2/V$. (10)

4. Answer any three

a) Determine the radius of curvature at the origin to the curve $x^4 + y^2 = 6a(x+y)$.

b) Find the asymptotes of $x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$.

c) If $f(x,y)=0$ be the equation of a curve, show that the radius of curvature at any point (x,y) to the curve can be expressed as $\rho=\dfrac{\left(f_x^2+f_y^2\right)^{3/2}}{f_{xx}f_y^2-2f_{xy}f_xf_y+f_{yy}f_x^2}$.

d) Using Lagrange's method of undetermined multiplier compute the minimum distance of the plane x + y+ z =1 from the origin. (10)

SECOND HALF

Answer any three questions

(Two marks are reserved for general proficiency)

5. (a) Evaluate

$$\int\int_R \frac{1}{\sqrt{x^2+y^2}}dxdy,$$

where $R=\{|x|\le 1, |y|\le 1\}$.

(b) Evaluate

$$\int\int\sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}dxdy$$

over the positive quadrant bounded by the circle $x^2+y^2=1$.

(5+6)

6. (a) Test the convergence of the series

$$\frac{6}{1.3.5}+\frac{8}{3.5.7}+\frac{10}{5.7.9}+\dots$$

(b) Test the convergence of the series $\sum a_n$, where $a_n=(n^3+1)^{\frac{1}{3}}-n$.

(6+5)

7. (a) Prove the series

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\dots+(-1)^{n+1}\frac{x^n}{n}+\dots \text{ to } \infty$$

is absolutely convergent when $|x|<1$ and conditionally convergent when $|x|=1$.

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}.$$

(6+5)

8. (a) Find the equations of the tangent plane and normal line to the surface $2x^2+y^2+2z=3$ at the point (2,1,-3).

(b) Find the directional derivative of $\vec{v}^2$, where

$$\vec{v}=xy^2\hat{i}+zy^2\hat{j}+xz^2\hat{k}$$

at the point $(1,2,1)$ in the direction of the normal to the sphere $x^2+y^2+z^2=6$

at the point (2,1,1).

(5+6)

9. (a) Show that,

$$\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational. Find a scalar function ϕ such that $\vec{A} = \vec{\nabla}\phi$.

(b) Verify Stokes' theorem for

$$\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k},$$

where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(5+6)