

SUBJECT: Reliability-based Analysis and Design (CE 803/4)

Use separate answer scripts for each half. All notations and abbreviations used have their usual meanings. Assume reasonable data, if not given. Answer any three Questions from each half. All question carry equal marks. Two marks are kept for neatness in each half.

FIRST Half

1. A simply supported RC beam of span 8m is subjected to deterministic dead load of intensity 3 kN/m, random live load of mean intensity 6 kN/m and SD of 3 kN/m. The allowable mean concrete strength is 30.28 N/mm² and SD=4.54 N/mm². The allowable mean strength of steel is 320 N/mm² and SD=32 N/mm². Assuming all the random variables as normally distributed, obtain the reliability index by FOSM. Given, d=300 mm, b=250mm and A_{st}=1000 mm². The resistance of the beam can be

taken as:
$$R = f_y A_{st} d \left[1 - \frac{0.77 f_y A_{st}}{b d f_{ck}} \right]$$

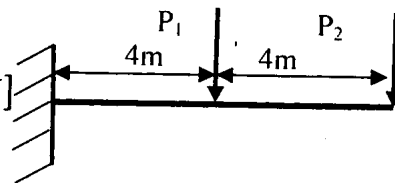
2. A simply supported steel I beam of length is subjected to a concentrated load P at centre. The thickness of web t_w = 1.25mm. The allowable shear stress is R and the depth of section is d and these are uncorrelated normal random variables with following given data: μ_P = 4000 N, σ_P = 1000 N, μ_R = 95 N/mm², σ_R = 10 N/mm², μ_d = 50 mm, σ_d = 2.5 mm. Find the reliability index by FORM method.

3. A tension member is subjected to an axial load, P. The allowable tensile stress is R and the diameter of the circular cross section is d. All these parameters are uncorrelated normal random variables with following given data: μ_P = 5000 N, σ_P = 2000 N, μ_R = 280 N/mm², σ_R = 28 N/mm². It is further given that the nominal value of load is 5200 N and nominal value of allowable stress is 250 N/mm² and nominal value of diameter = 5.5 mm. Find the partial safety factors for a target reliability index of 4.0

4. (a) The moment carrying capacity of the beam of length is M. The external loads P₁, P₂ and capacity M are correlated normal random variables with following given data. Obtain reliability index.

Mean: $[\mu_M \quad \mu_{P_1} \quad \mu_{P_2}] = [250kN - m \quad 10kN \quad 10kN]$

Cov:
$$\begin{bmatrix} 900kn/m^2 & 0 & 0 \\ 0 & 9kN^2 & 6kN^2 \\ 0 & 6kN^2 & 9kN^2 \end{bmatrix}$$



(b) A load P acting on a structure is random and defined by Type I Extreme value distribution as following:

$$F_p(x) = \exp[-\exp\{-\alpha(x-u)\}]$$
 where, $\mu_p = u + \frac{0.577}{\alpha}$ and $\sigma_p = \frac{1.283}{\alpha}$

It is also given: μ_p = 8kN & σ_p = 2.0kN

Find the equivalent normal at mean point. Hence explain how such non-normal value can be tackled in FORM algorithm.

5. (a) Explain series system and parallel system with suitable examples. What is brittle and ductile element with regard to reliability evaluation of a system?

(b) Compute system reliability of the truss shown in Fig Q 5(b) with following given data assuming all random variables are uncorrelated Gaussian:

$$\mu_p = 12kN, \sigma_p = 1.2kN, \mu_{R1} = 16kN, \sigma_{R1} = 1.6kN, \mu_{R2} = 18kN, \sigma_{R2} = 1.8kN$$

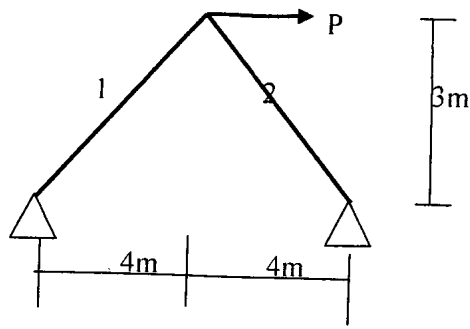


Fig. Q. 5(b)

SECOND half

6. (a) Explain FOSM with its limitations.
 (b) How you will find simple reliability bounds for series and parallel system?
 (c) How FORM is applied for correlated random variables case?
7. (a) In a fatigue reliability analysis of a butt welded joint the following data are observed. Fatigue strength coefficient $A=8 \times 10^{10}$ MPa, $C_A=0.4$, Stress range, $S_e=60$ MPa, slope of S-N curve $m=2.8$, $B=1$, $C_B=0.1$, Damage at failure, $\Delta=1.0$, $C_\Delta=0.32$. Estimate the reliability index and probability of failure for service life $N_s=2 \times 10^6$ cycles assuming a lognormal distribution for all the parameters.
 (b) A bridge can be damaged by failure in the foundation (F) and in the superstructure (S) with corresponding failure probabilities of 0.05 and 0.01, respectively. If there is a foundation failure probability that the superstructure will also suffer some damage is 0.50. i) What is the probability of the damage to the bridge? ii) If F and S are statistically independent, what is the probability of failure of the bridge? Represent the events in the Venn Diagram.
 (c) Use Rosenblueths' $2k+1$ point estimate method to calculate mean and coefficient of variation of $y=4x_1-x_2$, where x_1 is uniformly distributed between 5 to 15 and x_2 is normally distributed with mean 8 and standard deviation 1.5. Given, $\bar{y} = y_0 \prod_{i=1}^k \left(\frac{\bar{y}_i}{y_0} \right)$, $\delta = \sqrt{\left\{ \prod_{i=1}^k \left(1 + V_{y_i}^2 \right) \right\} - 1}$
8. (a) If (-1.061) be a uniformly distributed random number between 0 and 1, generate corresponding random number for a normal random variable with mean 12 and standard deviation 1. In general, what is the scheme to generate random variable with other nonnormal distributions?
 (b) How will you estimate probability of failure by Monte Carlo Simulation?
 (c) How will you estimate the number of simulations required to achieve a specified accuracy in direct Monte Carlo Simulation?
 (d) Explain Variance Reduction Techniques (VRT) for Simulation techniques? Discuss the fundamentals of Adaptive Sampling and Importance Sampling method.
9. (a) A Load is modeled by normal distribution with mean 200 kN and standard deviation 45 kN. Calculate the probabilities of load i) to be within 160 kN and 200 kN, ii) more than 200 kN. Also calculate the characteristic value of load at 95% confidence level.
 (b) Define 'Response surface model'. What are the different polynomial type response surface models? How can Response surface model help reliability evaluation?
 (c) Show design of experiments for CCD and 2^k Factorial design model for two standard normal variables using type III polynomial.
10. According to LRFD method compressive strength of a steel compression member is given by:
 $P_{cd} = 0.85 \times A \times f_{cr}$; $f_{cr} = 0.877 f_y / \lambda_c^2$ if $\lambda_c > 1.5$ and $f_{cr} = 0.658 \lambda_c^2 f_y$ for $\lambda_c \leq 1.5$,
 $\lambda_c = \left(\frac{KL}{r\pi} \right) \sqrt{\frac{f_y}{E}}$, $E=200\text{GPa}$, $f_y = 250\text{MPa}$, $A = 800\text{mm}^2$
 Obtain a RSM expression of P_{cd} as a function of KL/r using SD model with type-I polynomial. Take mean of KL/r as 131 and COV as 5%. Check the accuracy of the model.

A.3 STANDARDIZED NORMAL DISTRIBUTION FUNCTION

Table A.3 Standardized normal distribution function: a table of

$$F_U(u) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^u e^{-x^2/2} dx,$$

for $u = 0.0$ to 3.69

u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9482	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.8874	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

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