

Assume reasonable data wherever necessary

FIRST HALF

QUESTION NO. 1 COMPULSORY AND ANY TWO FROM THE REST

First Half

1. Figure 1 shows two rigid discs of moment of inertias  $J_1$  and  $J_2$  connected by a stepped shaft with torsional stiffnesses of two parts given by  $K_1$  and  $K_2$  as indicated in the figure. Neglecting the inertia of the shaft, determine the natural frequency of torsional oscillation of the system. (9)

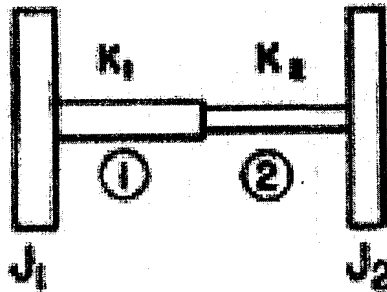


Figure 1

OR

- Figure 2 shows a single degree-of-freedom system. Determine the steady-state amplitude and phase of the motion of the mass with the harmonic input indicated in the figure. (9)

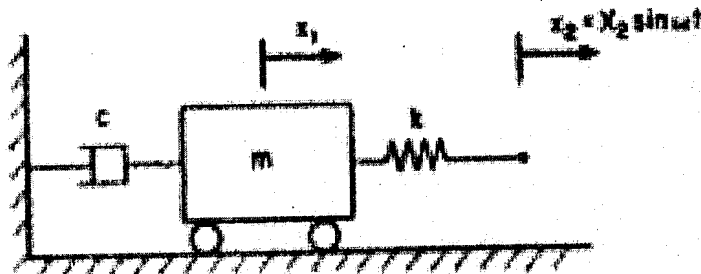


Figure 2

2. A rough casting of a rotor, with a mass of 200kg, is mounted on centres 110 cm apart for machining. The rotor is statically balanced by attaching two masses, one each in the planes A (9 Kg) and B (12 Kg), which are on opposite sides of the plane containing the mass centre of the rotor, at a distance of 50 cm and 35 cm, respectively, from it. The angle between the CG of the plane A and that of the plane B is  $90^\circ$ . The distance of each mass from the axis of rotation is 37 cm (for the plane A) and 44 cm (for the plane B). Determine the distance of the CG of the rotor from the axis of rotation, and its angular position with respect to the mass in the plane A. Also find the forces on the bearings when the rotor (with the masses attached in the planes A and B) runs at a speed of 100 rpm. (13)

3. In a four-cylinder engine, the centrelines the two pairs of cylinders form a V-angle of  $90^\circ$ . There are two cranks at  $180^\circ$ ; the distance between them is 10 cm. Each crank is connected to one pair of cylinders. The mass of the rcciprocating parts of each cylinder is 2 kg, the crank radius for each crank is 5 cm, and each connecting rod is 20 cm long. For the coordinate system shown in Figure.3, calculate the maximum primary and secondary components of the resultant unbalanced forces and moments when the engine is running at 1200 rpm. (13)

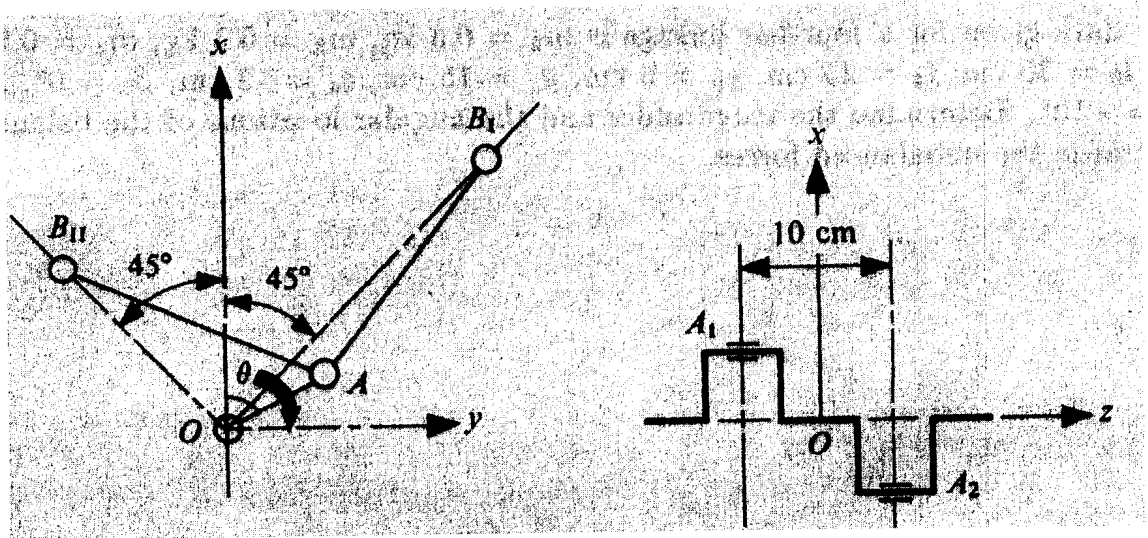


Figure 3

4. The arms of a Hartnell governor are of equal length. At midposition of the sleeve, the ball arm is vertical and the radius at which the ball rotates is 8.25 cm when the equilibrium speed, neglecting friction, is 450 rpm. On changing the speed by 1%, the governor is able to overcome the friction at this position. The friction force is assumed to have a constant value of 30 N at the sleeve. The sleeve moves  $\pm 1.6$  cm from the mean position. The minimum speed of the governor (including friction) is 428 rpm. The mass of the sleeve is 3.5 kg. Determine (i) the magnitude of the rotating masses, (ii) the spring stiffness, and (iii) the initial compression of the spring. (13)

5. The resisting torque on the crank of a riveting machine is shown in Figure 4. The duration of the cycle is 2 seconds. The motor driving the machine, however, has a speed of 1500 rpm and it delivers constant torque. The crankshaft of the machine is geared to the motor shaft. Neglecting frictional losses, determine
- the power of the motor, and
  - the moment of inertia of the flywheel mounted on the motor shaft to keep the speed fluctuation within  $\pm 2\%$  of the average speed.
- (13)

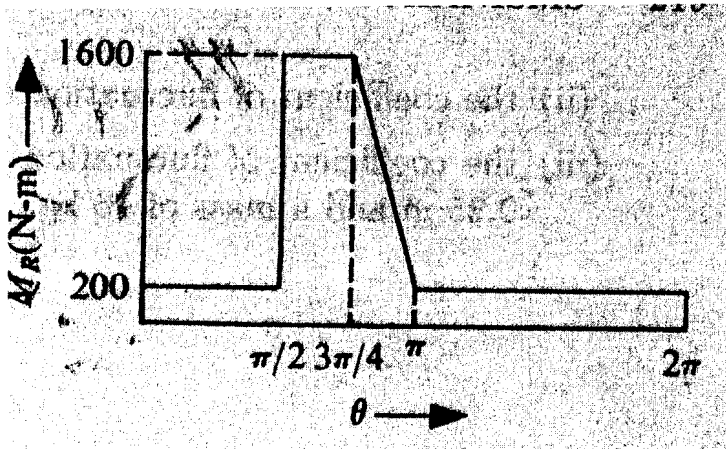


Figure 4

6. A trolley car with a total mass of 2700 kg runs on rails 1 m apart with a speed of 30 km/hr. The track is curved with a radius of 40 m towards the right of the driver. The car has four wheels each of diameter 70 cm and the total moment of inertia of each pair of wheels and the axle is  $15 \text{ kgm}^2$ . The car is driven by a motor running in the direction opposite to that of the wheels at a speed five times the speed of rotation of the wheels. The motor and the gear pinion have a moment of inertia  $10 \text{ kgm}^2$ . The rails are at the same level and the height of the CG of the car is 1 m above the rail level. Determine the vertical force exerted by each wheel on the rails.
- (13)

## Second Half

### ANSWER QUESTION NO. 7 AND ANY TWO FROM THE REST

7. (a) Figure 5 shows a uniform beam of length  $L$  mass per unit length  $m$  and flexural stiffness  $EI$ . The left end of the beam is pinned and the right end is clamped as shown. Starting from the beam function, determine the frequency equation for free vibration. (7)
- (b) For the beam shown in Figure 6 use the following assumed polynomial mode  $X_a(x)$  satisfying all the boundary conditions. Using Rayleigh's method, determine the first natural frequency using the symbol given in part (a). (8)

$$X_a(x) = \sum_{i=0}^4 \alpha_i x^i$$

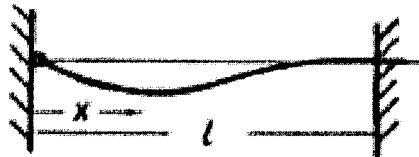


Figure 5

8. (a) Consider an undamped primary system  $(k, m)$  under harmonic excitation  $F_0 \cos \omega t$ . The system is found to be resonating. An absorber  $(k_a, m_a)$  tuned to the excitation frequency is attached to the primary system. Determine the two natural frequencies of the resulting system in terms of the mass ratio  $\nu = \frac{m_a}{m}$ . (6)
- (b) In part (a), the forcing frequency is 1500 cpm. If the two natural frequencies have to lie outside the range 1280 - 1800 cpm, what should be the minimum value of the mass ratio? (4)
9. (a) Consider an undamped spring-mass system  $(k, m)$ . Obtain the impulse-response function  $g(t)$  for this system. (4)
- (b) Explain how the function  $g(t)$  can be used to obtain the response for any arbitrary excitation  $F(t)$  and now use the result of part (a) to obtain the indicial response (response to unit step input)  $h(t)$  for the system considered in part (a). (6)
10. Assuming the mass and stiffness matrices are symmetric for a multi degree-of-freedom system, derive the orthogonality conditions of the normal modes. What is the physical significance of these orthogonality conditions and how these are used to solve the forced vibration problem? (10)