Bengal Engineering and Science University, Shibpur

B.E. 5th Semester (Aerospace Engineering) Final Examinations, December 2012

Numerical Methods and Computational Tools (AE 503)

Full Marks: 70 **Time: 3 hrs**

Answer any five(5) questions
All questions are of equal value

1.(a) Using direct method, derive the element stiffness matrix of a uniform truss element, arbitrarily oriented in the x-y plane.

(b) What are the properties of global stiffness matrix? Explain in brief, different approaches of imposing boundary conditions within global stiffness matrix.

2.(a) What are shape functions? Describe its properties.

(b) Write down the shape functions with shape diagrams of (i) two noded linear element (ii) three noded quadratic bar element (iii) four noded bilinear plane element.

3.(a) Given $\frac{dy}{dx} = 1 + y^2$, where y = 0 when x = 0. Using 4th order Runge-Kutta method, compute y(0.2), y(0.4), y(0.6). Using ABM method compute y(0.8).

(b) Explain with example, the differences between IVP & BVP.

4. Solve the BVP, $\frac{d^2y}{dx^2} - y = 0$ with y(0) = 0 and y(2) = 3.62686, taking step size 0.2. The exact solution being $y = \sinh x$, compute the error in each step.

5. Solve, using Crank – Nicholson scheme, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the initial condition, $u = sin\pi x$ at t = 0 for $0 \leq x \leq 1$ and the boundary conditions u = 0 at x = 0 and x = 1 for t > 0. Take h = 0.2, λ = 1 and compute the values of u at the internal mesh points up to two time steps.

6. How the Gauss quadrature is different from closed Newton-Cotes quadrature? Compute $GL_3(f)$ for the integral $\frac{1}{\pi}\int_0^{\pi} \cos(0.6\sin(t))dt$

7. Use the recursive trapezoidal rule to compute the approximations T(0), T(1), T(2) and T(3) for the integral $\int_1^5 \frac{dx}{x}$. Also compute the sequential approximations S(1), S(2), S(3) and B(2), B(3) for the same integral. In each case, compute the amount of deviation from the exact result of the integral.

8. Consider the following systems of equation: $\left.\begin{array}{r} -2x + y + 5z = 15 \\ 4x - 8y + z = -21 \\ 4x - y + z = 7 \end{array}\right\}$. Solve using Jacobi method and Gauss-Siedel method, taking same initial guess and for 5 iterations.