INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR
B.E. 4TH SEMESTER (AE) FINAL EXAMINATIONS, 2014
Viscous Fluid Flow (AE 401)

Time: 3 hrs | Full Marks: 70

(i) Answer any **five** questions
(ii) **Do not** write anything on this question paper

---

1. Consider steady incompressible flow of a constant-property Newtonian fluid within the annular gap between two infinitely long concentric cylinders of radii R_1 (inner) and R_2 (outer). The cylinders rotate in the same direction with angular velocities ω_1 (inner) and ω_2 (outer). Solve the Navier-Stokes system of equations to determine the radial distribution of the tangential velocity. Also, find the ratio ω_1/ω_2 for which the flow is irrotational. **11+3**

2. a) For flow past a solid sphere, using dimensional arguments show that the drag-coefficient becomes independent of Re_d (Reynolds number) at large Re_d (> 1000). **4**

   b) Discuss the techniques of delaying boundary layer separation. **6**

   c) Consider two-dimensional stagnation point flow on a flat plate; use the results of scale analysis to show that the boundary layer thickness remains constant along the wall (x direction). Plot the qualitative variation of wall shear stress with x. **2+2**

3. Derive the equation for conservation of linear momentum for a newtonian incompressible flow in the x-direction. State and apply all the assumptions required. **11+3**

4. Consider steady, incompressible, two-dimensional, laminar boundary layer flow along a flat surface. Assume a polynomial of the fourth degree for the velocity function in terms of the dimensionless distance from the wall η $(= y/\delta)$, i.e.

$$\frac{u}{U} = f(\eta) = a + b\eta + c\eta^2 + d\eta^3 + e\eta^4,$$

   applicable in the range $0 \le \eta \le 1$. Here, U is the velocity of the uniform flow at upstream.

   a) Utilize the following boundary conditions to determine the constants a, b, c, etc.

$$\text{at } y=0:\ u=0,\ \frac{\partial^2 u}{\partial y^2}=0; \qquad \text{and} \qquad \text{at } y=\delta(x):\ u=U,\ \frac{\partial u}{\partial y}=0,\ \frac{\partial^2 u}{\partial y^2}=0$$ **3**

   b) Using Von Karman momentum integral relation estimate the variation of local boundary layer thickness δ/x, and displacement thickness δ^*/x with local Reynolds number (Re_x). Also, determine the total skin-friction drag on one side of plate if its length and width are L and b, respectively. **8+3**

5. A flat plate of essentially infinite width and breadth is suddenly accelerated to a constant velocity U_0 in its own plane beneath a viscous fluid. The fluid is at rest far above the plate. Making appropriate assumptions set up the governing differential equation and boundary conditions for finding the velocity field u in the fluid. Transform the equation into an ordinary differential equation and solve it. **5+9**

Contd. to page 2

6. a) Consider steady fully-developed flow through a circular tube. Derive Darcy's equation for frictional head loss. Is this equation valid in turbulent flow? **6+1**

   b) Applying angular momentum conservation to an infinitesimally small control volume (rectangular parallelepiped), show that stress tensor is symmetric. Clearly mention all the assumptions involved. **6+1**

7. a) Consider steady, incompressible, two-dimensional, laminar boundary layer on a flat plate subjected to zero pressure gradient. Show that the dimensionless velocity profiles will be self-similar at sufficiently far downstream of the leading edge only if

$$\frac{U_\infty \delta}{\nu}\left(\frac{d\delta}{dx}\right) = \text{constant}$$

   where, U_∞ is free stream velocity, ν is kinematic viscosity, and δ is *characteristic thickness* of boundary layer. Reduce the boundary layer equation into an ordinary differential equation and express the boundary conditions in terms of similarity variables. **9**

   b) A jet plane having a wing area of 17.2 m^2 flies at a speed of 685 km/h when its engines develop 7350 KW. 65% of this power is used to overcome the aerodynamic drag. If the drag is 31.5% of the total lift force generated by the wings, find (i) the total weight of the plane, (ii) C_D and C_L. (assume air density = 0.72 kg.m^3) **5**

Important relations/equations in Cylindrical Coordinates

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

Convective time derivative:

$$\mathbf{V}\cdot\nabla = v_r\frac{\partial}{\partial r} + \frac{1}{r}v_\theta\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}$$

Laplacian operator:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

The r-momentum equation:

$$\frac{\partial v_r}{\partial t} + (\mathbf{V}\cdot\nabla)v_r - \frac{1}{r}v_\theta^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r + \nu\left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)$$

The θ-momentum equation:

$$\frac{\partial v_\theta}{\partial t} + (\mathbf{V}\cdot\nabla)v_\theta + \frac{1}{r}v_r v_\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + g_\theta + \nu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right)$$

The z-momentum equation:

$$\frac{\partial v_z}{\partial t} + (\mathbf{V}\cdot\nabla)v_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z + \nu\nabla^2 v_z$$

where

$$\epsilon_{rr} = \frac{\partial v_r}{\partial r} \qquad \epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)$$

$$\epsilon_{zz} = \frac{\partial v_z}{\partial z} \qquad \epsilon_{\theta z} = \frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}$$

$$\epsilon_{rz} = \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \qquad \epsilon_{r\theta} = \frac{1}{r}\left(\frac{\partial v_r}{\partial \theta} - v_\theta\right) + \frac{\partial v_\theta}{\partial r}$$

Viscous stress components:

$$\tau_{rr} = 2\mu\epsilon_{rr} \qquad \tau_{\theta\theta} = 2\mu\epsilon_{\theta\theta} \qquad \tau_{zz} = 2\mu\epsilon_{zz}$$

$$\tau_{r\theta} = \mu\epsilon_{r\theta} \qquad \tau_{\theta z} = \mu\epsilon_{\theta z} \qquad \tau_{rz} = \mu\epsilon_{rz}$$

Angular-velocity components:

$$\omega_r = \frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}$$

$$\omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}$$

$$\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta}$$