

M.Sc. (Applied Physics) 2nd Semester Examination, 2010

Electromagnetic Theory
(PGP-201)

Time : 4 hours

Full Marks : 100

Answer any FIVE questions.

1. What are retarded potentials corresponding to a charge distribution with a charge density ρ ?

Show that the retarded potential (ϕ) is a solution of inhomogeneous Helmholtz equation $\nabla^2 \phi = -\rho/\epsilon_0$ where the symbols have their usual meanings.

Justify the expression for four potential.

$$A^\mu = \left(\frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{R} d\tau' - \frac{1}{4\pi\epsilon_0 c} \int \frac{\mathbf{J}(\mathbf{r}') \cdot \mathbf{R}}{R^2} d\tau' \right)$$

where $S = R - R'$, $\beta = v/c$, \mathbf{r} = position coordinate from source point to field point. Write down the corresponding expressions for Lienard Weichart potentials.

(2+8+8+2]

2. a) Considering a relativistic charged particle with colinear velocity and acceleration, show that the power radiated per unit solid angle is given by

$$\frac{dP}{d\Omega} = \frac{c^2}{16\pi^2 \epsilon_0} \frac{p^2 \sin^2 \theta}{(1 - \beta \cos \theta)^5}$$

where all symbols have their usual meanings.

From the above, also show that the angle (θ_0) at which radiation is maximum (when $v \rightarrow c$) is given by,

$$\cos \theta_0 = \beta \quad \text{where } \beta = \frac{v}{c} \quad \text{and } \Lambda = \frac{\Lambda'}{c}$$

- b) Derive the expression for angular distribution of radiation for a charged particle in circular motion. Discuss how angular distribution varies when $\beta=0$ and $\theta=90^\circ$, for a circular accelerator.

(6+2)+(8+4)]

3. Calculate the differential and total scattering cross-section for a bound electron which

experiences a damping force equal to $\frac{e^2 v}{6\pi \epsilon_0 c^3} \mathbf{r}$, when a linearly polarised plane

electromagnetic field is incident on it. Compare the scattering crosssection with that of Thomson's scattering. Explain at what condition, the phenomenon of Rayleigh scattering becomes predominant.

{14+3+3]

4. a) Consider a metallic rectangular waveguide filled up with a pure dielectric material, in which an electromagnetic wave is propagating along z-direction. Solving Maxwell's equations prove that for TM_{mn} -mode :

$$E_z(x, y, z) = A \sin k_x x \cdot \sin k_y y \cdot e^{-\gamma z}$$

where the notations carry usual meanings.

- b) Show that the waveguide exhibits the property of a high pass filter.
 c) Derive and explain the significance of the time-average power Poynting vector of $TM_{3,2}$ -mode. [12+3+5]

5. a) (i) Find out the non-zero electric and magnetic field components of the TE-mode in a parallel plate waveguide filled up with a dielectric material.

(ii) Show that the ionosphere acts as a dispersive medium.

- b) (i) Consider a current distribution localized in a small region of space. Find the magnetic field at a point far away from the region of current distribution.

(ii) A charge distribution localized in a region is placed in an external electric field. Write down the energy of the system in terms of contributions from different multipoles. [6+4)+(6+4]

6. a) (i) Write down the action function of a charged particle moving in an external electromagnetic field.

(ii) Obtain equation of motion in covariant form by applying Hamilton's variational principle.

(iii) Find the Canonical momentum of the particle.

(iv) Write down Hamiltonian for the charged particle.

- b) Write down the component of electromagnetic field tensor. Use Lorentz transformation property to obtain electric and magnetic field vector in a frame moving with velocity v with respect to frame where the magnetic field vanishes.

- c) What is dual tensor to F^μ_ν ? Why $F^\mu_\nu F^{\mu\nu}$ can not be used as Lagrangian density for electromagnetic field even if it is a Lorentz scalar.

[(2+4+2+2)+(2+6)+2]

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7. a) (i) Construct the Green function for a sphere in electrostatic boundary value problem.
- (ii) Consider a conducting sphere of radius 'a' is made up of two hemispheres separated by a small insulating ring. The potential on the surface of the upper hemisphere is $+v$ while that on the surface of the lower hemisphere is $-v$. Find the potential at a pt. inside and outside the sphere.
(You need not perform explicitly the integral in the expression).
- b) Find the change in energy when a dielectric object with linear response is placed in an electric field whose sources are fixed.
- c) A spherical cavity of radius a is made in a dielectric medium with dielectric constant ϵ . An external field E is applied along z direction say. Find electric field inside the cavity. [(3+7)+6+4]