BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

P.G. (Mechanical Engg.) 1st Semester Examination, 2011

Subject: Finite Elements in Mechanical Systems (ME-929)

Full Marks: 70 Time: 3 Hours

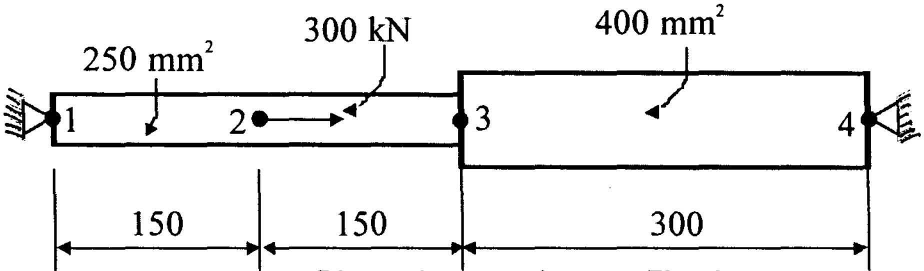
Answer FIVE questions taking at least TWO from each group

Group - A

1a) Derive the equations of a 1-D linear bar element subjected to concentrated end loads and body force along the axis of the bar. Write potential energy of the element in standard form to get the element equation.

b) Consider the bar shown in Fig. 1. Determine the nodal displacements at nodes 2, 3. There is no body force in the bar. Use three linear elements and take E = 200GPa.

[7+7 = 14]



Dimensions are in mm, Fig. 1

2a) What are the advantages of weak form of an weighted integral statement?

b) The following differential equation arises in connection with heat transfer in a rod insulated round the circumference:

$$-\frac{d}{dx}\left(k\frac{dT}{dx}\right) = Q \text{ in } 0<x<L, \text{ subject to } T(0) = T_0,\; k\frac{dT}{dx} + h(T - T_a) = 0 \text{ at } x = L.$$

The symbols have their usual meaning; T_a is the ambient temperature to which the right end of the bar is exposed. Derive the weak form of the governing equation and using Galerkin's procedure, obtain the element equation. Using two linear elements, estimate temperatures at x=L/2 and at x=L. Take L = 0.1m, Q = 0, k = 10.0 $Wm^{-1}°C^{-1}$, h = 25 $Wm^{-2}°C^{-1}$, T_0 = 50°C, T_a = 5°C. [3+11 = 14]

3a) From FEM point of view, what is the difference between a bar element and a beam element?

b) Compute the transverse deflection and slope at the right end (node 3) of the beam shown in Fig. 2. Choose elements as per your convenience. The right end is supported by a linear spring. No derivation is necessary. [3+11 = 14]

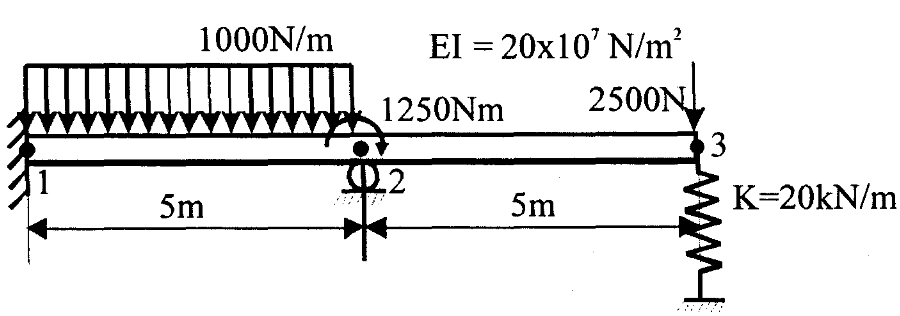


Fig. 2

The stiffness matrix of a typical beam element of length L and flexural rigidity EI may be taken as

$$[k_e]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

4a) A 4-node quadrilateral element having global coordinates (1,1), (5,1), (6.6) and (1,4), taken in order of connectivity, have the displacement vector $[0,\ 0,\ 0.2,\ 0,\ 0.15\ 0.1,\ 0,\ 0.05]^T$ respectively. The coordinates and the displacement components are in mm. Find the u, v displacements of a point P in the element whose location in the parent element is given by the natural coordinates r=0.5 and s=0.5.

b) Using the 2x2 Gauss-Quadrature formula, evaluate the integral $\iint_A (x+y)dxdy$, where A denotes the element mentioned in part (a). Show the calculation steps clearly but no derivation is necessary. W(1) = W(2) = 1.0; [r(1), r(2)] = ±0.577; [s(1), s(2)] = ±0.577.

[6+8 =14]

Group – B

5) For the two dimensional body, shown in Fig. 3, determine the temperature at nodes 1 and 2. The bottom and the left sides are insulated while the top side is maintained at a temperature of 500°C. The right side is subjected to heat transfer by convection. Take T_a = 20°C, and h = 20 W/(m^2-°C). There is no heat generation in the body and the front and back surfaces are also insulated. Thermal conductivity k = 10 W/(m-°C). Derivation is not compulsory. [14]

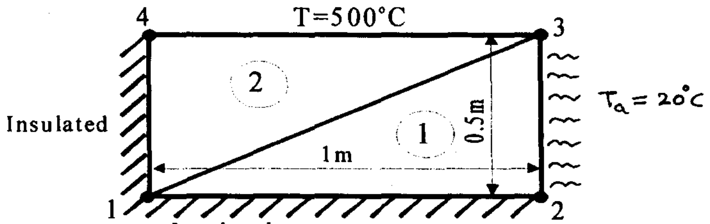


Fig. 3

6) From the FE equation of a beam element, how do you develop the FE equation of a straight member with axial load and transverse loads simultaneously. Now develop the element equation of a member of a plane frame oriented at an angle of α to the +X axis, with length L, cross-sectional area A and flexural rigidity EI. [14]

7) For the plane stress element shown in Fig. 4, determine its element equation. The connectivity of the element is i-j-k. The element has a pressure of magnitude 2000 unit perpendicular to the side j-k and is subjected to a temperature rise of 30°. Take E = 30E6, υ = 0.25, t = 1.0, α = 7E-6. Symbols have their usual meaning and all the values are in consistent units. No derivation is necessary. [14]

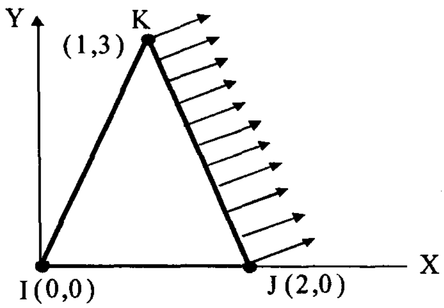


Fig. 4

8a) What do you understand by convergence in FE analysis? What do you understand by completeness in the approximation of the dependent variable?

b) The convection terms of the stiffness matrix for a 2-D heat transfer element are of the form $k_{ij} = \int_A hN_iN_j dA$, where h is the convection coefficient (constant value) and A is the element area. Derive the shape functions N_2 and N_4 of a conventional six node triangular element and compute the stiffness element k_{24}.

[6+8 = 14]