PG (ME) 1st Semester Final Examination 2011
Advanced Conductive and Radiative Heat Transfer (ME 901)

Time 3 hrs Full Marks 70

Answer all questions
All questions carry equal marks

1. The general heat conduction equation in Cartesian coordinates is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_G}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Using coordinates transformation formulae from Cartesian coordinates to spherical coordinates, derive the following general equation of heat conduction in spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\frac{\partial T}{\partial \vartheta}\right)+\frac{1}{r^2\sin^2\vartheta}\frac{\partial^2 T}{\partial \varphi^2}+\frac{q_G}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

2. (a) Find the steady state temperature distribution in an infinite slab of width 2b in which heat is generated at a uniform rate of q_G units per unit volume. The heat transfer coefficient at the surface is h and the ambient temperature is T_f. What is the expression of maximum temperature?
(b) A dam 5 m high, 10 m long and of average thickness 1 m, is made by pouring concrete. The hydration of concrete (k = 1.05 W/mK) results in an evolution of heat equal to 58 W/m^3. Consider the dam to be an infinite slab and calculate the maximum temperature in the concrete if the ambient temperature is 35°C and the surface heat transfer coefficient is 11.6 W/m^{2}K.

3. (a) Find an expression for the two-dimensional steady state temperature distribution in the rectangular plate. The three edges x = 0, x = a and y = 0 are maintained at a temperature T = 0, while the fourth edge y = b is maintained at a temperature T_g. Neglect temperature variation in the z-direction. There is no heat generation.
(b) A long rectangular steel bar (cross-section 10 cm × 5 cm) is kept in an environment such that the two 10 cm faces are maintained at 30°C, one 5 cm face is at 30°C and the other is at 130°C. Find the steady state temperature along the centre line of the bar.

4. A solid steel ball 5 cm in diameter and initially at a temperature of 450°C is quenched in a controlled environment whose temperature is maintained at a steady value of 90°C. Given: h = 115 W/m^{2}K, ρ = 8000

kg/m^3, C_p = 0.42 kJ/kgK, k = 46 W/mK. Determine the time taken by the centre of the ball to reach a temperature of 150°C if (a) internal temperature gradients are neglected, and (b) internal temperature gradients are not neglected. Compare the results in (a) with that in (b) by finding the Bi. Assume the following:

$$\lambda_1 R = 0.430 \quad \textbf{and} \quad \frac{2(\sin \lambda_1 R - \lambda_1 R \cos \lambda_1 R)}{\lambda_1 R - \sin \lambda_1 R \cos \lambda_1 R} = 1.01865$$

5. **State Planck's law and Wien's law of radiation. Derive the Stefan-Boltzmann equation by integrating Planck's law.**