ME-ICE, Part I, 1st Semester Final Semester Examination, December 11
Paper Code: Information Theory and Coding (ICE 902)

Full Marks: 70 Time: 3 Hrs.

(Answer any **Five** questions)

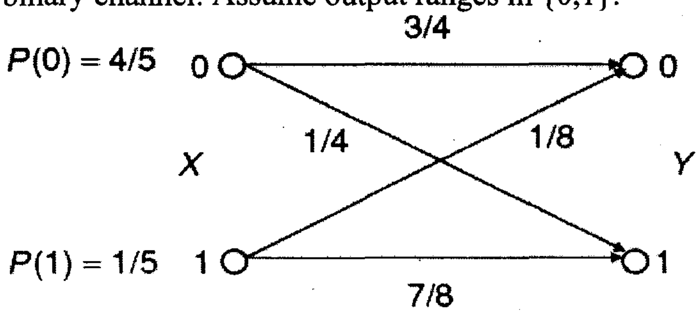
1. (a) Draw the entropy curve for two sources emitted from a discrete memoryless source. Find the maximum entropy.
(b) Calculate entropy for the source emitting following symbols with the probabilities tabulated below.

xi	Pi
A	½
B	¼
C	1/8
D	1/20
E	1/20
F	1/40

(c) If each alphabet in the above table is encoded with fixed length encoding and emitted per second, what will be the minimum information rate?
(d) Find the amount of information contained in the massage ABABBA, FDDFDF.

5 + 2 + 3 + 4 = 14

2. (a) Find source entropy H(X) and output entropies H(X/0) and H(X/1) for the following binary channel. Assume output ranges in {0,1}.



(b) Let X be a random variable that adopts values in the range A = $\{x_1, x_2, \ldots, x_M\}$ and represents the output of a given source. Show that, $0 \leq H(X) \leq \log_2(M)$

10 + 4 = 14

3. (a) Perform Huffman encoding on the following eight messages with M = 8 having probabilities given as 0.46, 0.15, 0.10, 0.10, 0.07, 0.05, 0.03, 0.02. Calculate average code length and code efficiency. Draw the tree diagram of encoding.
(b) What do you mean by decodability of a source decoder?
(c) State an advantage of variable length encoding over fixed length encoding.

8 + 4 + 2 = 14

4. For a (7, 3) code, a generator matrix is,

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find systematic encoding for the code. Draw the hardware diagram for encoding.
(b) Determine parity check matrix for the above code.
(c) For the following generator matrix can you perform systematic encoding?

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

If the generator polynomial is written as $\mathbf{g(x)} = \mathbf{1} + \mathbf{x}^2 + \mathbf{x}^3 + \mathbf{x}^4$ find the codeword for the message d_2.

8 + 2 + 4 = 14

5. (a) For the (15,ll) binary Hamming code with generator $\mathbf{g(x)} = \mathbf{x}^4 + x + \mathbf{1}$ For the message polynomial $\mathbf{m(x)} = \mathbf{x} + \mathbf{x}^2 + \mathbf{x}^3$, find systematic code polynomial codeword.
(b) For $\mathbf{c(x)} = \mathbf{1} + \mathbf{x} + \mathbf{x}^3 + \mathbf{x}^4 + \mathbf{x}^5 + \mathbf{x}^9 + \mathbf{x}^{10} + \mathbf{x}^{11} + \mathbf{x}^{13}$, determine the message polynomial if nonsystematic encoding is employed.
(c) What are the advantages of cyclic encoding over linear block coding?

4 + 6 + 4 = 14

6. (a) Consider the following convolutional transfer function matrix.

$$G_b(x) = \left[1 \quad \frac{1+x+x^2}{1+x^2}.\right].$$

(i) Draw the encode circuit
(ii) For the message polynomial $\mathbf{m(x)} = \mathbf{1} + \mathbf{x} + \mathbf{x}^2 + \mathbf{x}^3 + \mathbf{x}^4 + \mathbf{x}^8$, determine the code polynomials and the coded output sequence.

(b) Draw the multiplication hardware for a 5[th] order generator polynomial g(x).

4 + 6 + 4 = 14

7. (a) Consider the division of the polynomial $\mathbf{d(x)} = \mathbf{x}^8 + \mathbf{x}^7 + \mathbf{x}^5 + \mathbf{x} + \mathbf{1}$ by $\mathbf{g(x)} = \mathbf{x}^5 + \mathbf{x} + \mathbf{1}$. Show the computation steps for long division and design the hardware for division.
(b) Find all the conjugacy classes of $GF(2^4)$ with respect to GF(2).

8 + 6 = 14