Bengal Engineering and Science University, Shibpur

M. E. (ETC) 1st Semester Final Examination, 2011

Discrete Structure

(M-902)

Time: 3 hours **Full Marks: 70**

Answer any **SIX** questions, taking **THREE** from each half. The questions are of equal marks. 2 Marks are reserved for general proficiency in each half.

FIRST HALF

1. (a) A relation R on the set Z (the set of all integers) is defined in the following way, $\{R = (a, b) \in Z \times Z : a - b \text{ is divisible by } 7\}$. Show that R is an equivalence relation. Find all the distinct equivalent classes of the relation R.

   (b) Is the mapping $f : R \to (-1, 1)$ defined by $f(x) = \frac{x}{1+|x|}$ a bijective mapping? Justify your answer.

2. (a) Show that the set Z of all integers does not form a group under the binary operation * defined as x * y = x - y, for every x, y belong to Z.

   (b) Prove that the intersection of any two sub-groups of a group $(G, *)$ is again a sub-group of $(G, *)$.

   (c) Show that a group G is abelian, if $(ab)^2 = a^2b^2$ for $a, b \in G$.

3. (a) Define a cyclic group. Prove that every sub-group of a cyclic group is cyclic.

   (b) Prove that the set of all real numbers of the form $(a + b\sqrt{2})$, where a, b are rational numbers, is a field under usual addition and multiplication.

4. (a) Prove that every field is an integral domain.

(b) Define a Boolean algebra. Prove that in a Boolean algebra B, for all a in B

$$a + a = a \quad \text{and} \quad a.a = a$$

5. (a) In a Boolean algebra B, $a,\ b,\ c \in B$, reduce the following Boolean function to its disjunctive normal form:

(i) $(a + b + c).(ab + ac)$
(ii) $(a + b)\ (a + b')\ (a' + c)$.

(b) Construct the switching circuit representing

$$ab + ab' + a'b'$$

and show that the circuit is equivalent to the switching circuit $(a + b')$.

M- 902 :

Group- B
(Answer any three)
(Symbols have their usual meanings)

6. a) Show that the umber of internal vertices in a binary tree is one less than the number of pendent vertices.
b) Show that a graph is a tree if and only if there exists exactly one path between every pair of vertices.

7. a) Show that any connected graph with n vertices and (n-1) edges is a tree.
b) Show that every connect graph has at least one spanning tree. With the help of an example show that it is possible to have more than one spanning trees.

8. a) Show that every cutest in a connected graph G must contain at least one branch of every spanning tree.
b) Prove that with respect to a given spanning tree T, a chord c_i that determines a fundamental circuit γ occurs in every fundamental cutest associated with the branches in γ and in no others.

9. a) What is the planner graph? Show that $e \leq 3n - 6$ is a necessary condition for planarity.
b) With the help of an example, establish that the above condition is not sufficient.

10. a) What is the chromatic polynomial? Find it for a tree of n vertices.
b) With the help of truth tables show that De Morgan's laws are satisfied in propositional calculus.
c) Obtain a DNA form of $P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$.