Bengal Engineering and Science University Shibpur

Aerospace & Applied Mechanics Department

End Semester Examination: Fall Semester 2011-12

AM 927: Theory of Vibration I

Time Allowed: 3hrs | **Full Marks: 70**

Answer any five questions

Q1(a) Derive the equation of motion of the block (Fig.1) in terms of *x* and its derivatives. (7)

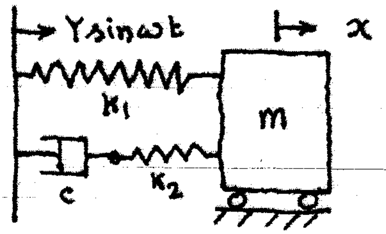


Fig. 1

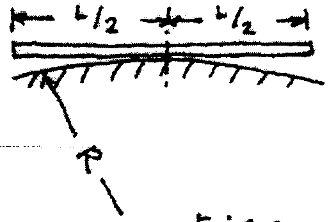


Fig. 2

(b) A thin uniform rigid wooden plank of length *l* and mass *m* is resting on a cylindrical surface with radius *R* as shown in Fig. 2. Find out the frequency of natural rocking oscillation of the plank when slightly disturbed from its equilibrium position. Friction between the plank and the surface is enough to prevent any slip. (7)

Q2(a) Figure 3 shows a very simplified model of a car travelling along a wavy road with 5 cm as the amplitude of the waviness and 50 m as the wave length. The mass of the car body is 1500 kg. If the suspension can be represented by a single spring of stiffness *k* find out the value of this stiffness *k* so that the amplitude of vertical osci--llation of the car body does not exceed 2 cm at a speed of 60 km/hr. (7)

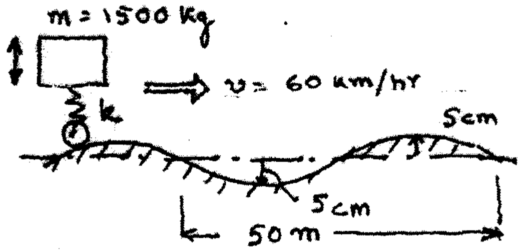


Fig. 3

(b) A simple spring-mass system shown in Fig. 4a is acted upon by a force *F(t)* as indicated in Fig. 4b. Taking a step size of 0.5 sec plot the response *x* vs. *t*, the time, for 4 sec and determine the amplitude of residual oscillation. What is the nature of the residual oscillation? (7)

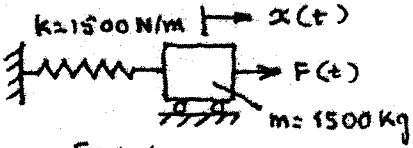


Fig. 4a

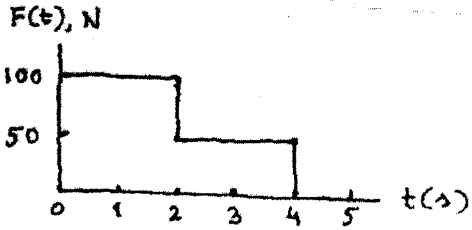


Fig 4b

Q3(a) If {Xi} and {Xj} be two natural modes of an n degree of freedom system with [M] as the diagonal mass matrix then prove that

$$[X_i][M]\{X_j\} = 0 \; ; \; i \neq j \qquad (4)$$

(b) A simple spring-mass system is shown in Fig. 5a. The block of mass m is acted upon by a periodic force $F(t)$ the nature of which is indicated in Fig. 5b. Find out the steady state

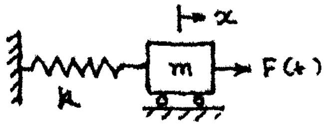


Fig. 5a

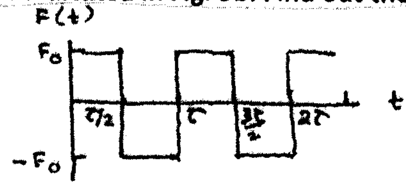


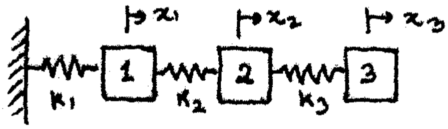
Fig. 5b

response $x(t)$ of the block. (10)

Q.4(a) A seismic vibration pick up for measuring velocity has a natural frequency of 7.5 Hz and a damping factor $\zeta = 0.5$. Find out the lowest frequency a vibration can have that can be monitored by the pick up if the allowed error be 2.5%. (7)

(b) Figure 6 shows an un damped 3 degree of freedom system where the numbers indicate the masses of the respective blocks in appropriate unit. The first two natural modes of free vibration of the system are

$$\{X^{(1)}\} = \begin{Bmatrix} 0.39522 \\ 0.75864 \\ 1.00000 \end{Bmatrix}, \quad \{X^{(2)}\} = \begin{Bmatrix} -1.4210 \\ -1.6070 \\ 1.0000 \end{Bmatrix}$$



Find out the third mode. $\{X^{(3)}\}$

Fig. 6 (7)

Q.5 Derive Rayleigh's Quotient for estimating the upper bound value of the natural frequency of vibration of a bar as shown in Fig. 7. Estimate the natural frequency of longitudinal vibration of a uniform prismatic bar of material density ρ, cross sectional area A and length l with its both ends fixed.

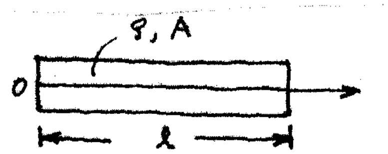


Fig. 7

(14)

Q.6 Derive the differential equation of transverse vibration of a uniform beam with flexural rigidity modulus EI, material density ρ and length l. Find out the natural frequencies and mode shapes when both the ends of a beam are simply supported. (14)

Q.7

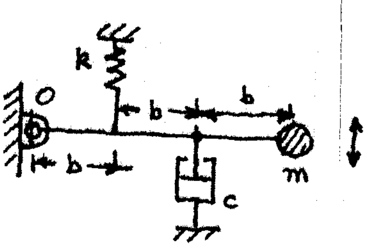


Fig. 8

Figure 8 shows an oscillating system consisting of a mass m attached to the free end of a rigid light rod of length $3b$ hinged at O. The rod is suspended by a spring of stiffness k as shown and a viscous dashpot is attached in the manner indicated in the figure. Find out the expression for the critical damping coefficient of the dashpot (14)

IMPORTANT: ALL DRAWINGS AND ANSWERS SHOULD BE UNAMBIGUOUS AND SELF EXPLANATORY.