Bengal Engineering and Science University, Shibpur.
First Semester Master of Engineering Examination - 2011
Theory of Elasticity and Plasticity ( AM – 903 )

Full marks : 70 Time : 3 hours

Answer any five questions.
All questions carry equal marks

1(a) Starting from fundamentals , deduce expressions for strain components $\varepsilon_x$ , $\varepsilon_y$ , $\gamma_{xy}$ in terms of the displacements u , v at any point of a 2 D stressed body . Draw neat explanatory sketches.

(b) Show that the compatibility condition for 2D problems in terms of stress components , in absence of any body force , can be expressed as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\sigma_x + \sigma_y\right) = 0$$

(c) Using the stress-strain relationship and the equations of equilibrium, show that in absence of body forces, the displacements in problems of plane stress must satisfy

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) = 0$$

and a companion equation of a similar form.

2(a) Explain what is a stress function. Bring out its importance in solving elasticity problems.

(b) A cantilever beam of length l , depth 2c and unit thickness is loaded by a downwards concentrated force P at the free end. Using appropriate stress functions, derive expressions for stresses at any point ( x,y ) and compare the solution with those obtained by the elementary theory of strength of materials.

3(a) Draw a 3 D square element with right handed system of coordinate axes and show all normal and shear stresses using conventional nomenclature.

(b) Prove the compatibility equations in polar coordinates in the form

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}\right) = 0$$

4(a) Obtain the equilibrium equations in polar coordinate system in the following form

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial_r - \partial_\theta}{r} + R = 0$$

$$\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = 0$$

(b) A cylindrical vessel of internal and external radii a and b respectively is subjected to internal and external pressures $p_i$ and $p_o$. Work out expressions for radial and circumferential stresses $\sigma_r$ and $\sigma_\theta$ If the cylinder is subjected to internal pressure only, what are the values of the maximum radial and circumferential stresses and where do they occur ?

5(a) A concentrated force P is applied on a straight boundary of an infinitely large horizontal plate. P represents load per unit thickness and the distribution of the load along the thickness is uniform. Find what stress components $\sigma_x$, $\sigma_y$, $\tau_{xy}$ does it produce at a distance r from the point of application of the load and at an angle θ with the vertical through the point of application of the load.

(b) A circular disc of diameter d is subjected to two opposite compressive forces P along the vertical diameter. Find the expression for $\sigma_y$ at any point on the horizontal diameter and hence find the maximum value of the stress with its location.

6(a) Explain with illustrations what are meant by plane stress and plane strain problems. Give explanatory sketches whenever necessary.

(b) A small circular hole of radius a is located in the middle of a plate of small thickness and large width , the plate being subjected to a uniform tension in the longitudinal direction. Show that the maximum tensile stress occurring at the periphery of the hole at points located at the ends of a diameter perpendicular to the applied stress is three times the uniform stress applied.