BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

M.E. (Appl. Mech/MET) Part I FINAL EXAMINATIONS, 2011

Advanced Mathematics and Statistics for Engineers

M-901

Time: 3 hours **Full Marks: 70**

Use separate answer script for each half.
Answer **SIX** questions, taking **THREE** from each half
Two marks are reserved for general proficiency in each half.

FIRST HALF

1. Solve the following PDEs: ($p = \partial z / \partial x$, $q = \partial z / \partial y$)

   (a) $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$.

   (b) $\cos(x + y)p + \sin(x + y)q = z$. (5+6)

2. (a) Solve the following non-linear PDE: ($p = \partial z / \partial x$, $q = \partial z / \partial y$)

   $z^2(p^2 z^2 + q^2) = 1$.

   (b) Solve the following second order PDE: ($D \equiv \partial / \partial x$, $D' \equiv \partial / \partial y$)

   $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{3x+4y}$. (5+6)

3. (a) Obtain the PDE by eliminating the arbitrary constants a and b from $z = a(x + y) + b$.

   (b) If a slightly flexible string is stretched between two fixed points and the motion is started by drawing aside through a distance 'b' at a point on the string distance 1/5th of the length 'l' of the string of one end. Show that the displacement 'y' at any time 't' is given by $y = \dfrac{25b}{2\pi^2}\sum_{n=1}^{\infty}\dfrac{1}{n^2}\sin\left(\dfrac{n\pi}{5}\right)\sin\left(\dfrac{n\pi x}{l}\right)\cos\left(\dfrac{n\pi ct}{l}\right)$. (2+9)

4. (a) Form a PDE by eliminating the arbitrary functions f and g from $z = f(x + iy) + g(x - iy)$, where $i^2 = -1$.

   (b) An insulated rod of length 'l' has it's ends A and B maintained at 0^o C and 100^o C until steady state conditions prevail. If the end A is raised to 20^o C and the end B is reduced to 80^o C; find the temperature at a distance x from the end A at time t. (2+9)

5. (a) Suppose a cup of tea initially at a temperature of 180^{o}F, is placed in a room which is held at a constant temperature of 80^{o}F. Suppose that after one minute the tea has cooled to 175^{o}F. What will be the temperature after 20 minutes?

(b) Using Cobweb analysis discuss the stability of the critical points of the difference equation $N_{t+1} = \frac{3}{4} N_t + 10$. (6+5)

6. (a) Find the general solution of the following equation:

$$x_{t+2} - 5x_{t+1} + 6x_t = 4^t + t^2 + 3.$$

(b) What is Beverton-Holt Stock recruitment curve, why this curve is called compensatory? Discuss the stability analysis of the Beverton-Holt model. (5+6)

SECOND HALF

7. (a) State and prove Baye's theorem.

(b) In a bolt factory machines A, B, C manufacture respectively 20%, 30% and 50% of the total. Of their output 10%, 5% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machine B ?

(a) A four digit number is constructed using the digits 0, 1 , 2 , 3, 4, 5, 6. What is the probability that the constructed number is greater than equal to 3000?

4+4+3 = 11

8. (a) The mean yield for one-acre plot is 600 Kgs with S.D. 50 Kgs. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots would you expect to have yield (i) over 700 Kgs (ii) below 550 Kgs and (iii) what is the lowest yield of the best 100 plots? [Given that $P(0 < Z < 1.28) = 0.4$, where $Z \sim N(0,1^2)$]

(b) Define Moment Generating Function (MGF) of a random variable X. Find the MGF of Binomial variate $X(n, p)$ and hence find its mean and variance.

6+5=11

9. a) The joint *p.d.f.* of two random variables X and Y is given by:

$$f(x,y)=\frac{9(1+x+y)}{2(1+x)^4(1+y)^4}; \quad 0\le x<\infty,\ 0<y<\infty$$

Find the marginal distribution of X and Y, and the conditional distribution of Y for X=x. Also find (i) $P(X>1)$, (ii) $P(X<Y\,|\,X<2Y)$

b) What do you understand by correlation between two random variables? The random variables X and Y are jointly normally distributed and U & V are defined by:

$$U = X\cos(\alpha) + Y\operatorname{Sin}(\alpha)\ ; \quad V = Y\cos(\alpha) - X\operatorname{Sin}(\alpha)$$

Show that U and V will be uncorrelated if $\tan(\alpha)=\frac{2\rho\sigma_X\sigma_Y}{\sigma_X^{\,2}-\sigma_Y^{\,2}}$, where the symbols have their usual meaning.

5+6=11

10. (a) Define critical region, Errors of Type - I & Type - II and power of Test.

(b) If $W=\{x:\ x\ge 1.5\}$ is the critical region for testing the hypothesis $H_0:\theta=2$ against the alternative hypothesis $H_1:\theta=1$, on the basis of the single observation from the population,

$$f(x,\theta)=\theta\exp(-\theta\, x)\ ,\quad 0\le x<\infty,$$

obtain the probability of Type - I & Type – II errors and power of the test.

(c) Define unbiased estimator. Show that $\frac{\sum_{i=1}^{n} x_i\left(\sum_{i=1}^{n} x_i - 1\right)}{n(n-1)}$ is an unbiased estimate of θ^2, for the sample $x_1, x_2, \ldots\ldots\ldots x_n$ drawn on X which takes the value 1 or 0 with respective probabilities θ and $(1-\theta)$.

2+5+4=11

11. (a) State and prove Neyman-Pearson Lemma.

(b) A population is defined by the p.d.f. $f(x,\theta)=(1+\theta)x^{\theta}$, obtain Maximum Likelihood Estimator (MLE) of the unknown parameter θ on the basis for sample of size n.

6+5=11