BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR
M.E. 1ST SEMESTER (CE) FINAL EXAMINATIONS, 2011
Advanced Numerical Methods and Computer Programming (CE 920)

Full Marks: 70 **Time: 3 hrs**

Answer any five questions.

1. Given a set of three points (x_1,y_1), (x_2,y_2) and (x_3,y_3), derive a second order Lagrange interpolation polynomial that passes through these points. Also, write a program that constructs the Lagrange interpolation polynomial for a given set of data points and then computes the interpolation value at a specified value. (14)

2. (a) Discuss the improvement in the Gauss-Seidal method over the Jacobi Iteration method for the solution of linear equations. Also give an algorithm for the Jacobi Iteration method for the solution of n linear equations. (7)

   (b) Solve for steady-state temperature in a steel plate of size 15 cm x 15 cm if two adjacent sides are held at 0°C and the other two sides are held at 50°C. (7)

3. (a) Write a program to obtain the linear regression coefficients for a given set of data. (7)

   (b) Determine coefficients of characteristic polynomial for the eigenvalue problem:

$$\begin{bmatrix} 2 & -1 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \lambda^2 \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} \quad (7)$$

4. (a) Solve $d^2y/dx^2 + 2\,dy/dx - y = 2x$ with y(0)=0, y'(0)=1 using fourth-order Runge-Kutta method. (8)

   (b) Write a program for solution of ordinary first order differential equation using the fourth-order Runge-Kutta method. (6)

5. (a) Obtain Newton's interpolation polynomial to pass through the points (1,1), (3,5) and (6,10) and then modify the polynomial for an additional point (5,9). Use the modified polynomial to estimate the value of y when $x = 4.2$. (7)

   (b) Estimate relative errors in (i) $z = 0.4172\text{x}10^4\text{-}0.4171\ \text{x}10^4$ (ii) $z = \sqrt{1+x}$, $x = 4$ considering 4 digits mantissa system. (4)

   (c) The relative error in $w = x^n$ is n times the relative error in x. Justify. (3)

6. (a) Solve the following system of equations by using Dolittle decomposition method.

$$\begin{aligned} 1.5x_1 + x_2 + 0.5x_3 &= 5 \\ x_1 + 1.5x_2 + x_3 &= 7 \\ 0.5x_1 + x_2 + 1.5x_3 &= 7 \end{aligned} \quad (7)$$

(b) Derive the Newton-Raphson formula using the Taylor Series expansion. (3)

(c) State a few precautions to improve accuracy in computations. (4)

7. (a) Compare the following two direct methods for the solution of n linear equations

   i) Basic Gauss Elimination method
   ii) Gauss Elimination with pivoting (4)

   (b) What is Cholesky's factorization? (3)

   (c) Write a computer program for integration using composite Simpsons 1/3 rule. (7)